

MEDIEVAL LOGIC AND
METAPHYSICS

A MODERN INTRODUCTION

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§2 ANSELMIAN REGRESSES*

A tactic sometimes employed in philosophical argument involves showing that the position of one's opponent commits him to an infinite regress. On the assumption that the regress in question is unacceptable, such a consequence refutes the opponent's argument. A process of this sort can most obviously be used to suggest superfluity in the terms of a definition, as in the following example from St Anselm's dialogue on paronymy (*De Grammatico*):

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|----|--|---|
| .1 | TUTOR. . . . if <i>literate</i> is <i>man displaying literacy</i> , then wherever <i>literate</i> appears the words <i>man displaying literacy</i> may be correctly substituted for it. | MAGISTER. . . . <i>si grammaticus est homo sciens grammaticam ubicumque ponitur grammaticus, apte ponitur homo sciens grammaticam.</i> |
| | STUDENT. That's right. | DISCIPULUS. <i>Ita est.</i> |
| | T. Hence if it is appropriate to say, 'Socrates is a literate man', it is equally appropriate to say, 'Socrates is a man displaying literacy man'. | M. <i>Si igitur apte dicitur: Socrates est homo grammaticus, apte quoque dicitur: Socrates est homo homo sciens grammaticam.</i> |
| | s. So it follows. | D. <i>Consequitur.</i> |
| | T. But every man displaying literacy is a literate man. | M. <i>Omnis autem homo sciens grammaticam est homo grammaticus.</i> |
| | s. Yes. | D. <i>Ita est.</i> |
| | T. Thus Socrates, who is a man displaying literacy man, is a literate man man, and since a literate is a man displaying literacy, it follows that Socrates is a man displaying literacy man man, and so on to infinity. . . . It | M. <i>Socrates igitur qui est homo homo sciens grammaticam, est homo homo grammaticus. Et quoniam grammaticus est homo sciens grammaticam, consequitur ut Socrates sit homo homo homo sciens grammaticam,</i> |

* It would be unfortunate if the comparatively dense nature of this section's subject-matter and mode of presentation were to make it into a *pons asinorum*, obstructing advance into the later sections. Readers who experience undue difficulty herein may pass immediately to the later sections, each of which can be read as a unity largely independent of the others.

has now been sufficiently proved that *literate* does not signify man.

s. Quite sufficiently.

t. What then is left?

s. It can only signify . . . displaying literacy.

et sic in infinitum. . . . Iam satis probatum est quia non significat hominem.

D. *Satis.*

M. *Quid ergo restat?*

D. *Ut non significet aliud quam scientem grammaticam.*

(HDG 4.2414, 4.31)

The Tutor is here arguing, following a pattern suggested by Aristotle, that an account of the meaning of *grammaticus* 'literate' should not comprise a mention of its constant referent, i.e. man. He uses the regress which results from such a mention in order to refute the Student's contention.

The aim of the present section is to sketch the general nature of the field of Anselm's dialogue and then to inspect in some detail the logical significance of yet another argument from regress which occurs in the same work.

Grammaticus 'literate' is for Anselm an example of what are known as denominative names or paronyms. It is a familiar fact of current English that most adjectives have a corresponding abstract noun, as 'white' has 'whiteness', 'smooth' has 'smoothness', 'literate' has 'literacy', and so on. The concrete members of such pairs serve as examples of the paronyms with which we are now concerned. They receive their title of 'paronym' or its alternative 'denominative name' because of their supposed derivation from their corresponding and partly equiform abstract nouns (cf. *HL* §3.123, *HDG* ch. IV, *HW* and *HAN*). The problem of the signification of paronyms lies essentially in the question as to whether an account of their *meaning* should at the same time involve an account of their *reference*. If one concentrates on the adjectives which constitute the main body of the paronyms, then, as H. W. B. Joseph says, 'their invention implies the consideration of some quality or character in the thing in abstraction from the rest of the thing's nature' (*JL* 38). Under such circumstances one is tempted to say that the corresponding abstract noun is a sufficient account of their meaning, and any mention of the things to which they happen to refer is superfluous (*HL* §3.131). Such appears to have been the position of the logical tradition of which Anselm was the inheritor (*HL* §3.124). On the other hand, if one considers certain paronyms which constantly happen to qualify only a single species of being, as in the case of Anselm's 'literate', which is used only of human beings, then the result might be the demand that some mention

of this constant referent should be included in any account of what this and other paronyms signify. This sort of demand would be backed up in Anselm's own day by the thesis inherited from the grammarians, in particular Priscian, to the effect that *all* names (of which adjectives were considered a species) signify both a substance (e.g. *man*, in the case of 'literate') and a quality (e.g. literacy); HDG §4.109, HL §3.124. J. S. Mill, with his doctrine that all 'terms' have both connotation and denotation, appears to be somewhat on the side of Priscian (HDG §4.314).

In pursuance of the logical, rather than the grammatical tradition, Anselm holds that there *is* a difference between the ways in which adjectival words and substance-words signify (HDG §5.2, HL §3, HAN). Any account of the meaning of paronyms (which are now assumed to exclude substance-words such as 'man') ought to depict them as being completely open as regards the type of object to which they may apply. Thus, although as a matter of fact 'literate' has always been *used* in respect of human beings, this should not affect the account given of its *meaning*, if 'meaning' is taken in a strict technical sense (HL §2.13.1, HDG 4.232–2.2341). This is the background to the regress-argument quoted at .1 above, and Anselm's conclusion at this juncture could be expressed:

- .2 *grammaticus est habens grammaticam* (HDG 4.31, 4.700)
 a literate is a haver of literacy
 or
 literate is . . . having literacy

Anselm strains all his powers of explanation to suggest that something like the second English version of .2, with its gapped (i.e. functorial) formulation, is a true representation of the thesis he intends to express. Thus he expels by means of various arguments (of which .1 is one) the *homo* 'man' part from the Student's suggested definition of *grammaticus* 'literate' as *homo habens* (or *sciens*) *grammaticam* 'man having (or displaying) literacy' (HDG 4.31). Again, as we are to see below, while accepting, in a parallel case, that *albus est aliquid habens albedinem* '(a) white is something having whiteness' is a truth, he will nevertheless insist that *albus est habens albedinem* 'white is . . . having whiteness' is the proper expression of what he intends to say. The expulsion of even the colourless *aliquid* 'something' from the meaning of 'white' shows that any clue as to the nature of the referent of the word which may be given by the gender (in this case masculine) of the Latin version of the word must be considered irrelevant; cf. HDG §2.32. This expulsion corresponds, of course, to the removal of 'man' which is arrived at in .1.

The issue can be further contextualised and stated thus: it was

pointed out in I §3 that decisions on the appropriateness of definitions framed according to the canons of Aristotle and Boethius can involve resource to statements such as *grammaticus est species* 'literate is a species' (HDG 4.2411), *grammaticus est qualitas* 'literate is a quality' (HDG 4.603), and *homo est genus* 'man is a genus' (HDG 4.2411), in which the *est* 'is' is plainly peculiar and problematical in so far as it is not the same *est* 'is' as we have in *Socrates est philosophus* 'Socrates is a philosopher'. In terms of Leśniewski's Ontology at least some of these problem-sentences may be elucidated in terms of the verb-flanked '. . . is . . .' defined at II §5.19 (cf. LAS 248–9 and I §3.4 above). Such an interpretation, with its incomplete, functorial arguments for the '. . . is . . .' (and for functors defined in terms of that '. . . is . . .') is at any rate one which allows for that openness of meaning for paronyms which Anselm so urgently requires, as indicated by .1 and the contentions summarised at the close of the last paragraph. It is perhaps with some intuition of the special logical grammar which his thesis requires that Anselm, here dealing with a question of definition, holds that .2 can yield assertions such as *grammaticus est grammatica* 'literate is literacy' which, though required in this definitional context, do not cohere with *usus loquendi* 'ordinary language'. In this respect (i.e. in having consequences not cohering with ordinary usage) .2 would be related to the problematical sentences quoted above and which involve open verb-like arguments for their '. . . is . . .'; these are the sort of arguments which Anselm appears to require for the *est* 'is' of .2. The Student, on the other hand, insists on interpreting assertions such as .2, notwithstanding their occurrence in the context of definitional discussion, as being of a more familiar sort, i.e. as involving an *est* 'is' having two *nominal* arguments (as does 'Socrates is a philosopher'); cf. HL §3.221. The Student's claim that the referent 'man' should be included in the definition of 'literate' appears to be an index of his insistence on the non-verb-like, merely nominal, interpretation of the completions of the *est* 'is' of .2, an interpretation which is coverable by the first of the two English translations which accompany .2.

What we may here be faced with, therefore, are two possibilities for the interpretation of names or name-like expressions: they may indeed be names (like the 'man' of 'Jack is a man') but sometimes are better construed as verb-like functors (like the 'man' of '*man* is a species'). This contrast comes out well when the statement:

- .3 *albus est (idem quod) habens albedinem*
 (a) white is (the same as) a haver of whiteness
 or
 white is (the same as) . . . having whiteness

is discussed (*HDG* 4.8). For the Tutor this is in the present context of the same type as .2, in that it resembles the latter throughout except in so far as the paronym *albus* 'white' has replaced *grammaticus* 'literate'; as an alternative exemplification of his thesis he therefore accepts it. However, the Student insists on interpreting .3 as involving an 'is' whose two arguments are nominal (cf. the first English translation annexed to .3) and the Tutor shows that this supposition leads to an infinite regress. Now should it turn out that an analysis of this regress confirms in detail Anselm's intuition that .2 and .3, interpreted as having non-nominal functorial arguments for their 'is' (cf. the second English translation annexed to .3), need not give rise to regress, then the conjecture that he appreciates the unusual semantical categories involved in the situation will be to some extent confirmed. Here now is the discussion which centres around .3:

4. STUDENT. As *white* is equivalent to *having whiteness*, it does not determinately signify this or that thing having whiteness, such as a body; rather it signifies indeterminately some thing having whiteness. This is because a white is either that which has whiteness or that which has not whiteness; but that which has not whiteness is not white, so that a white is that which has whiteness. Further, since everything which has whiteness must needs be something, a white must be something which has whiteness, or something having whiteness. Finally *white* signifies either something having whiteness or nothing; but nothing cannot be conceived to have whiteness, hence *white* must signify something having whiteness (cf. §4 below).
- DISCIPULUS. *Albus cum sit idem quod habens albedinem, non significat determinate hoc vel illud habens, velut corpus, sed indeterminate aliquid habens albedinem. Albus enim aut est qui habet albedinem, aut qui non habet. Sed qui non habet albedinem non est albus. Albus igitur est qui habet albedinem. Quare quoniam omnis qui albedinem habet non nisi aliquid est, necesse est ut albus sit aliquid quod habet albedinem, aut aliquid habens albedinem. Denique albus aut aliquid significat habens albedinem aut nihil. Sed nihil non potest intelligi habens albedinem. Necesse est ergo ut albus significet aliquid habens albedinem* (cf. §4 below).

TUTOR. The question is not whether everything which is white is something, or whether it is that which has, but whether the word *white* contains in its signification the expression *something*, or *that which has*, in the way that *man* contains *animal*, with the consequence that in the same way as *man* is *rational mortal animal*, so also *white* is *something having whiteness* or *that which has whiteness*. Now many things are necessary to the being of anything you care to mention, and yet are not signified by the name of the thing in question. For example, every animal must be coloured as well as either rational or irrational, yet the name *animal* signifies none of these things. Hence although there is no white which is not something having whiteness or that which has whiteness, nevertheless *white* need not signify these facts.

Nevertheless, let us suppose that *white* can signify *something having whiteness*. Now *something having whiteness* is the same as *something white*.

s. It must be so.

t. *White* therefore always signifies *something white*.

s. Quite so.

MAGISTER. *Non agitur utrum omnis qui est albus sit aliquid aut sit qui habet, sed utrum hoc nomen sua significatione contineat hoc quod dicitur aliquid aut qui habet—sicut homo continet animal—ut quomodo homo est animal rationale mortale, ita albus sit aliquid habens albedinem aut qui habet albedinem. Multa namque necesse est rem quamlibet esse, quae tamen rei eiusdem nomine non significantur. Nam omne animal necesse est coloratum esse et rationale aut irrationale, nomen tamen animalis nihil horum significat. Quare licet albus non sit nisi aliquid habens aut qui habet albedinem, non tamen necesse est ut albus hoc significet.*

Ponamus enim quod albus sive album significet aliquid habens albedinem. Sed aliquid habens albedinem non est aliud quam aliquid album.

D. *Non potest aliud esse.*

M. *Albus igitur sive album semper significat aliquid album.*

D. *Ita sit.*

t. So that wherever *white* appears it is always correct to substitute *something white* for *white*.

s. That follows.

t. Hence when *something white* is used, the double expression *something something white* is also correct; when the double is correct, so also is the triple, and so on to infinity.

s. This is a derivable absurdity.

m. *Ubi ergo ponitur albus vel album recte semper accipitur pro albo aliquid album.*

d. *Consequitur.*

m. *Ergo ubi dicitur aliquid album, recte quoque dicitur bis: aliquid aliquid album; et ubi bis, ibi et ter, et hoc infinite.*

d. *Consequens et absurdum est hoc.*

(HDG 4.800–4.8120)

(The translation of *qui* 'who/that' has here been rendered in such a fashion as to give that neutrality after which Anselm is evidently striving, and which has been remarked on above; cf. HDG §2.32.) The contrasting logical levels presupposed by the Tutor and Student in this passage may be approached with reference to the Ontology described in Part II. In the first place, to the axiom and definitions given in II §5 one may add:

.5 $[\varphi\psi] : \cdot \varphi \circ \psi \cdot \equiv : [\chi] : \chi \in \varphi \cdot \equiv \cdot \chi \in \psi$

This defines a higher-order weak identity, analogous to II §4.3.12, but based on the 'ε' of II §5.19. It thus takes as its arguments the gapped functorial expressions which Anselm would seem to require (cf. .20, .21 below). It is at this level that we assume the Tutor to pitch his interpretation of .3.

Let us now consider the nominal expressions required for the expression of the Student's interpretation of .3. These will operate not at the level of the higher order 'ε' or 'ο' mentioned in the last paragraph, but rather at the level of the primitive, lower-order 'ε' which figures in the axiom of Ontology presented in Part II. If for present purposes we assume that abstract nouns are logically more akin to verbs than to nouns (an assumption to be developed in §5 and §6 below), then given the abstract noun *albedo* 'whiteness', hereunder abbreviated as 'ω', a corresponding nominal expression, i.e. *habens albedinem* 'whiteness haver' or *aliquid habens albedinem* 'something having whiteness' ('trm<ω>'), counterpart of the first English translation annexed to .3, can be introduced by means of II §5.15:

.6 $[a] : a \in \text{trm}\langle\omega\rangle \cdot \equiv \cdot a \in a \cdot \omega(a)$

Next *albus* 'white', the corresponding concrete form hereunder abbreviated as 'w', could be introduced thus:

$$.7 \quad [a] : a \in \mathbf{w} . \equiv . a \in \text{trm}\langle \omega \rangle \quad (\text{cf. } .6)$$

Definitions II §5.16 and II §5.15 may now also be used to give the sense of *aliquid album* 'something that is-white'; for the predicate 'is-white' (' $\in[\mathbf{w}]$ ') may be formed from the name 'w' thus:

$$.8 \quad [a] : \in[\mathbf{w}](a) . \equiv . a \in \mathbf{w} \quad (\text{cf. } .7 \text{ and II } §5.16)$$

and this predicate in turn used to form the nominal expression 'term satisfying is-white' (' $\text{trm}\langle \in[\mathbf{w}] \rangle$ '), i.e. *aliquid album* 'something white', thus:

$$.9 \quad [a] : a \in \text{trm}\langle \in[\mathbf{w}] \rangle . \equiv . a \in a . \in[\mathbf{w}](a) \quad (\text{cf. } .8 \text{ and II } §5.15)$$

Having thus made this excursion into some theorems of the theory of 'white' we are equipped to deal with the nominal expressions undergoing discussion in .4 in so far as they involve nominal arguments. (The more complex renderings of some of these nominal arguments (§6 below) are unnecessary for present purposes.)

As regards the Tutor's interpretation of .3 (the counterpart of the second of the English translations thereto annexed) it has already been noted (§1 of this part) that the functor ' $\text{Cl}[\]$ ', defined II §5.18, can be used to account for certain cases such as the ones with which we are concerned in his thesis, i.e. cases in which names or name-like expressions occur in the natural language, but have to be construed logically as being verb-like in character. Thus, to revert to an example already mentioned, the *homo* 'man' of '*homo est species*' 'man is a species' could in fact be rendered (using 'h' for *homo* 'man') as ' $\text{Cl}[\mathbf{h}]$ ' in order to fit it to become an argument of the higher-order *est* 'is' here involved, and which can be analysed as the higher-order ' \in ' (taking verb-like functors, not names as arguments) defined at II §5.19 above. (This suggestion stems from *LAS* 248–9; cf. also *HDG* §5.32, *HDG* §6.3126, *HL* §3.221, and *HAN*, as well as §6 below.) It is hence in terms of ' $\text{Cl}[\]$ ', also a suitable argument for the functor defined at .5, that the Tutor's contentions in .4 will be analysed below (cf. .19). It will also be useful to assume the following thesis, provable from the axiom and from the definitions quoted after it:

$$.10 \quad [ab] : \text{Cl}[a] \in \text{Cl}[b] . \equiv . \text{Cl}[a] \circ \text{Cl}[b]$$

(cf. II §5.18, §5.19 and .5 above.) This thesis shows us how, at this higher-order level, and with the arguments (' $\text{Cl}[\]$ ') now in question, an *est* 'is' amounts to an *est idem quod* 'is the same as'. This point is made in case anyone should have qualms about the use of the higher-order ' \circ ' to translate the higher-order 'is' (as at .19 below).

As a preliminary summary of .4 we may say that the Student opts for an interpretation of .3 such that the arguments of its *est (idem quod)* 'is (the same as)' are supposed to be unavoidably nominal. Forthwith the Tutor rejoins that such an interpretation, though not giving rise to falsehood, is nevertheless irrelevant in the present context, and shows that it leads to an infinite regress which his own (functorial and non-nominal) interpretation of those arguments avoids. This he construes as evidence for the rejection of the Student's interpretation (*HDG* 4.81).

In the detailed expansion of this summary which now follows the literary variants of *est (idem quod)* 'is (the same as)' which occur in the text of .4 (i.e. *non est aliud quam* 'is no other than' *recte semper accipitur pro* 'is always rightly taken in the place of', and so on) and their *de voce* 'utterance-related' alternatives (e.g. *significat* 'signifies') are hereunder uniformly replaced by the sign of weak identity, namely 'o' (cf. II §4.3.12). This would seem justified in view of the definitional context within which we are working. And although, in the text, it may be observed that Anselm carefully distinguishes between questions about meaning and questions about things, a full perusal of his dialogue will reveal that he would have no objection to the telescoping of these in the way which is implied by our use of 'o'.

Thus in .4 we find the Student giving to .3 the following interpretation:

- .11 *Album est aliquid habens albedinem*
A white (object) is something having whiteness

This would appear to be intended to bring out the nominal nature of the arguments of *est* 'is', and so to be of the form:

- .12 $w \circ \text{trm}\langle\omega\rangle$ (cf. .6, .7 and II §4.3.12, II § 5.15)

He then accepts

- .13 *aliquid habens albedinem* o *aliquid album*
something having whiteness o something white

This, in terms of the expressions made available above, may be interpreted as:

- .14 $\text{trm}\langle\omega\rangle \circ \text{trm}\langle\epsilon[\mathbf{w}]\rangle$ (cf. .7, .9)

Then from identities .11 and .13 the Student finds himself committed to the following further identity:

- .15 *albus* o *aliquid album*
white o something white

which amounts to:

.16 $w \circ \text{trm}\langle\epsilon[[w]]\rangle$ (cf. .7, .9)

As already remarked above, the gender of *albus* 'white' is here immaterial, so that the identity given in .16 can plainly be used to effect substitutions of *aliquid album* 'something white' wherever *albus* (or *album*) 'white' appears, even in .16 itself. One can hence infer:

.17 *aliquid album* \circ *aliquid aliquid album*
something white \circ something something white

The corresponding operation in the artificial language, i.e. the substitution of ' $\text{trm}\langle\epsilon[[w]]\rangle$ ' for ' w ' throughout .16 and in accordance with .16 itself, yields:

.18 $\text{trm}\langle\epsilon[[w]]\rangle \circ \text{trm}\langle\epsilon[[\text{trm}\langle\epsilon[[w]]\rangle]]\rangle$

The regresses initiated in .17 and .18 can plainly, by .15 and .16 respectively, be continued to infinity. It is on these grounds that the Tutor concludes that .11 must be rejected as an interpretation of the definition of *albus* 'white', formulated as at .3 above. In other words, the arguments of the *est (idem quod)* 'is (the same as)' of .3 must not be taken to be nominal in nature. Following the lines suggested above, the contrasting interpretation, involving open verb-like arguments (' $\text{Cl}[\]$ ') of the sort which the Tutor appears to require, could read:

.19 $\text{Cl}[[w]] \circ \text{Cl}[[\text{trm}\langle\omega\rangle]]$

Although this interpretation is the exact higher-order correlate of .12, the regress initiated at .16 cannot be reproduced at this level, even though .14 and .16 are brought in as well. This regress-evasion occurs because (as in the case of Anselm's Latin version) it is now impossible to contrive that the first argument of .19 becomes equiform with part of its second argument, whereas this equiformity is conceded in .16.

As remarked above, .19 is a higher-order version of .12. By the use of .22, which the Tutor accepts as a truth, one could infer from .19 the higher-order analogue of .14, namely:

.20 $\text{Cl}[[\text{trm}\langle\omega\rangle]] \circ \text{Cl}[[\text{trm}\langle\epsilon[[w]]\rangle]]$ (cf. .5)

From this, coupled with .19, one could then in turn infer:

.21 $\text{Cl}[[w]] \circ \text{Cl}[[\text{trm}\langle\epsilon[[w]]\rangle]]$

This last is the higher-order version of the regress-generating .16. Still, in this case an infinite regress cannot be generated by substitution of the second argument of the ' \circ ' of .21 for any unit of that second argument which is equiform with the first argument, since there is now no such equiformity. In this respect .21 differs from .16.

It looks, therefore, as though Anselm's intuition that .3 (and other definitions for like paronyms) must be expressed in non-nominal terms if a regress is to be avoided has been substantiated by the analysis provided above. One may well ask, however, whether this result is of any real logical or philosophical significance. The answer is that relative to the resources of Ontology it can be seen to have very little import. For .12, .14 and .16 can be shown to be inferentially equivalent to .19, .20, and .21 respectively. Hence given .21, inference of the regress from .16 is still possible. This regress is no more vicious than the repetition of 'p' which propositional calculus allows us to infer in ' $p \supset . p . p . p . . .$ ', for example.

From the point of view of the history of Logic, however, there is a lesson to be learned here. Nowadays we are in possession of systematic languages involving quantification over various types of variable for the expression of complex truths, including those required for definition. In the only slightly modified natural language of the medievals, however, recourse to a word such as *aliquid* 'something' was necessary for the expression of what we would construe as variables, and this was unreliable when exploited in contexts of any complexity (cf. §4 below). However, it is already evident that although truths such as .19 have a complicated structure which needs to be ultimately rendered in terms of the variables and quantifiers occurring in the appropriate definitions, they are nevertheless susceptible of a comparatively simple statement in what looks like natural language (e.g. .3). Hence, by insisting that formal, definitional discourse concerning the essences or 'forms' of things should involve recondite semantical categories (cf. I §3) with the sometimes consequently deviant language noted in passing in the present section, the medievals were enabled to keep their considerations on a suitably general plane without getting too much immersed in the details of quantifiers and variables with which their semi-artificial language was ill-equipped to deal. However, they lacked an efficient background system of logic in terms of which discourse at this formal level could be kept under control and explained. Thus the interrelations such as those revealed in the last paragraph between apparently conflicting theses could not be made explicit. The Student in .4, with his insistence on *aliquid* 'something', was not altogether wrong, after all. Such unnecessary mutual misunderstandings were then liable to give rise to the sort of confusion and argument at cross-purposes which we have witnessed and which are also touched upon in I §3 as well as in §5 of the present part.

The medievals, with their logical metaphysics and metaphysical logic, were working on lines which point rather in a contemporary direction, unlike some of the more disastrous efforts of intervening

'modern' philosophy. Unfortunately, they were working at about the same stage as that of a non-symbolic mathematics wherein it is necessary to remain at the level of expression exemplified in:

.22 The difference of two squared numbers is equal to the product of the sum of the two and the difference of the two.

instead of making use of variables, as in:

.23 $a^2 - b^2 = (a + b)(a - b)$

It is therefore a great pity if people now concerned (perhaps unwittingly) with projects akin to those of the medievals, still insist on remaining at the stage exemplified in .22 whereas (as the present work attempts to show) it is possible to go forward to improvements analogous to .23. Such an insistence can only result in an unnecessary continuation of the 'mock battle' picture of metaphysics, as described by Kant in the preface to the first edition of the *Critique of Pure Reason*.

§3 EXISTENCE AND INCLUSION

It is sometimes suggested that sentences of the traditional categorical form 'Every a is b ' (or 'All a is b ') should properly be analysed as being of the form, 'For all x , if $\varphi(x)$ then $\psi(x)$ ' (e.g. *RI* 162). As it stands this translation is unsatisfactory, since the predicate variables ' φ ' and ' ψ ' are not covered by any quantifier, and it hence has not the determinate sense possessed by the original of which it is supposed to be a translation; in fact, if one quantifies them universally, then the result is quite obviously too broad, since instances of ' φ ' and ' ψ ' could be just *any* predicates, as opposed to the ' \dots is a ' and ' \dots is b ' which would appear to be the ones appropriate in the case. But if one quantifies them particularly then the result is again too broad, since the exact predicates required as substituends for ' φ ' and ' ψ ' are not specified. In any case, with the restricted interpretation of the quantifiers, one now runs into the trouble described in Essay I of *QF*. It would therefore appear that a more satisfactory rendering of 'Every a is b ' (or 'All a is b ') is the one suggested by definition II §4.3.5, i.e.

.1 $[ab] : . a \subset b . \equiv : [c] : c \in a . \supset . c \in b$

Although this is certainly an improvement on the suggestion described above, since now any questions which may be raised concerning quantification over predicates no longer arise, and it is made clear that ' $\dots \in a$ ' and ' $\dots \in b$ ' are the required inner structures of ' φ ' and ' ψ ' respectively, it still has the disadvantage, described at length in