

Medieval and Renaissance Logic in Spain

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BLASIUS PELACANI, THE PARADOXES OF IMPLICATION AND THE NOTION OF LOGICAL CONSEQUENCE

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1. In recent years, it has almost turned into a commonplace —as some logicians seem now ready to assume— that 'the study of medieval logic provides an opportunity for interaction between contemporary philosophical reflections and the thought of our predecessors in the Middle Ages',¹ or conversely —as some historians seem prepared to admit— that most of the all too recent 'reconstructive scholarship' of medieval logical theories was 'never' due to 'a mere historical interest', but to 'a truly philosophical' one.² Accordingly, an examination of Blasius Pelacani's views on the paradoxes of implication —such as they are to be found in his *Questiones dialectice* on the *Tractatus* of Peter of Spain— would best seem to justify itself, if it could be brought to bear upon present-day discussions on the notion of logical consequence. However, the history of logical ideas turns out to be less trivial an endeavour than it might seem at first. Despite their proclaimed intentions, historians are often prone to take current theories for granted when they deal with medieval doctrines, and logicians tend to rely upon commonly accepted historical reconstructions when they look for an answer to their concerns. So historians have to admit that 'the distinction between metalanguage and objectlanguage', as it has been introduced by modern logicians, is sometimes 'handled too easily with respect to the medieval theories',³ and logicians cannot take much advantage of the historians' stock story that 'very many authors', including such diverse thinkers as the pseudo-Scotus and William of Ockham, all

¹ S. Read, 'Formal and Material Consequence, Disjunctive Syllogism and Gamma', in *Argumentationstheorie: scholastische Forschungen zu den logischen und semantischen Regeln korrekten Folgerns*, hrsg. von Klaus Jacobi, Leiden, Brill, 1993, p. 233.

² K. Jacobi, 'General Introduction', in *Argumentationstheorie*, p. xxiii.

³ R. van der Lecq, 'The Role of Language-levels in the Medieval Discussion on *Insolubilia*, in *Argumentationstheorie*, p. 277.

'subscribed to' a widespread 'general picture' of the distinction between formal and material consequence.⁴

All in all, logicians and historians both lean on unchecked authority: acquiescence to current logical views on the one hand and distorted historical reconstructions on the other mutually support themselves. Logicians accept reconstructions based on theories they want to revise and historians rely on notions they should help to modify. In these conditions, a proper examination of Blasius' two *questiones* on the paradoxes of implication can be severely hampered on both sides. A lack of common understanding on the very notion of logical consequence fails to ensure a well established theoretical background and the working historian is at a loss for tenable explanations, whereas preconceived historical accounts do not afford logicians a proper understanding of the medieval doctrines and deprive ongoing discussions of any possible suggestion. An unbiased approach should then imply a refashioned historical description as well as a fresh theoretical restart. Might we enable new logical solutions without modifying the historical account?

2. As a matter of fact, an accepted historical view on the theory of consequences, such as we find invoked by Ivan Boh, looks far from indisputable. According to Boh, an utterly 'novel if not revolutionary passage'⁵ is to be found in a 'commentary on the famous *Summulae* of Peter of Spain' written by one of Copernicus' teachers, John of Glogovia, a professor of logic and natural philosophy at the university of Cracow in the last quarter of the 15th century. In his book, as quoted by Boh, Glogovian advances the following claims:

From what has been said, the falsity of two rules laid down by the ancients (*antiqui*) is evident. The first rule is *From the impossible anything follows*, so that every proposition is true whose antecedent is impossible; for example, 'If Plato is a jack-ass, Plato is a stone'. The second rule is, *The necessary follows from anything*. Therefore every conditional is said to be true whose consequent is necessary, regardless of what the antecedent is; for example, they said that 'If Plato is a

⁴ Read, 'Formal and Material Consequence', p. 240.

⁵ I. Boh, 'John of Glogovia's Rejection of Paradoxical Entailment Rules', in *Die Philosophie im 14. Und 15. Jahrhundert: In memoriam Konstanty Michalski (1879-1947)*, hrsg. von O. Pluta, Amsterdam, Grüner, 1988, p. 373.

stone, man is an animal' is necessary, because for truth of a conditional it is required that the antecedent could not be true without the consequent.⁶

But is 'this passage' so 'surprising' and does it express 'a view that goes against the long logical tradition'?⁷ Is it really so new, or revolutionary? Moreover, how can we interpret Glogovian's 'positive characterizations' of true conditionals? In another passage quoted by Boh, John of Glogovia says:

The truth of a conditional is caused by the relationship (*ex habitu-dine*) of the antecedent to the consequent and not by the inherence of the predicate in the subject either in the antecedent or in the consequent as is caused in categorical propositions. Therefore every true conditional is necessary and every false conditional is impossible, because it is founded upon universal *intentions* such as *whole, part, genus*, and the like which are always either necessary or impossible. The reason is that with respect to every consequence the antecedent is related to the consequent as cause, or as cause of being, or at least as cause of following with respect to another, unless it has to it the relation of inclusion or containment.⁸

⁶ Ibid. Cf. *Super omnes tractatus parvorum logicalium Petri Hispani Magistri Johannis Glogoviensis*, alme florentissimeque universitatis Studii Kracouiensis, maioris Collegii artistarum, Leipzig, 1500 (no further reference to foliation is annexed to this quotation by Boh): 'Ex dictis patet falsitas duarum regularum ab antiquis positarum. Prima est, *Ex impossibili sequitur quodlibet* ut omnis conditionalis sit vera cuius antecedens fuerit impossibile; ut, 'Si Plato est asinus, Plato est lapis'. Secunda regula, *Necessarium sequitur ad quodlibet*. Ideo omnis conditionalis dicitur vera, cuius consequens est necessarium, quaecumque fuerit antecedens; ut, 'Si Plato est lapis, homo est animal', necessarium dicebant, quia ad veritatem conditionalis exigitur quod antecedens non possit esse verum sine consequente.'

⁷ Boh, 'John of Glogovia's Rejection', p. 373.

⁸ Ibid., p. 380-81. Cf. Glogoviensis, *Super omnes tractatus parvorum logicalium*, fol. 203v: 'Veritas conditionalis causatur ex habitu-dine antecedentis ad consequens et non ex inherentia predicati ad subiectum nec in antecedente nec in consequente, sicut causatur in categoricis. Quare omnis conditionalis vera est necessaria et omnis falsa est impossibilis, quia fundatur supra intentiones universales ut sunt *totum, pars, genus, et huiusmodi que sunt semper necessarie vel impossibiles*. Ulterius sequitur quod ex impossibili non sequitur quodlibet. Ratio quia in omni consequente antecedens se habet ut causa consequentis vel ut causa essendi vel ad minus ut causa consequendi respectu alterius, nisi ad ipsum habeat habitu-dinem includentis vel continentis.'

Here, 'what Glogovian means is', according to Boh, 'a connection' between the antecedent and the consequent 'which is somehow based on the *intentions* (meaning, concepts) whose relationship are recognized by topical maxims'. Thus, as a possible explanation, 'one might find it plausible to formulate topical maxims as if they were law-like sentences or rules and think of particular conditionals as "falling under" the law', but since 'we know that quantified logic is also truth-functional, and if a given antecedent is not satisfied, the corresponding conditional will automatically turn out to be true', in Boh's opinion 'we are already in the realm of paradoxes'.⁹ In other words, the topical solution is altogether discarded for not being a viable one.

On the face of it, though, both contentions do not seem to be fully supported by historical evidence. Logical inferences based on relevant conditionals seem to have been usually accepted by ancient logicians as a matter of course.¹⁰ The so-called Stoic 'unmethodically concluding arguments' and their counterparts discussed by late antiquity logicians, which were thought to be not 'formally valid' but 'materially valid arguments',¹¹ have been respectively compared by Jonathan Barnes to the *consequentiae formales* and *materiales* described by medieval authors¹² —and to our present purposes 'quite significantly' so.¹³ The "non-logical" rule of inference *by which* the conclusion is correctly but unmethodically derived' in an unmethodical argument 'is just one of the premisses *from which* the same conclusion is both correctly and methodically derived' in a methodical argument. But if an unmethodical argument 'is not' to be called 'a truncated version' of a corresponding methodical argument,¹⁴ 'the answer' is to be found in 'the logical status of the proposition that acts in turn as a missing premise and as an as-

⁹ Ibid. p. 381.

¹⁰ Cf. D. Buzzetti, 'On Proclus' Comparison of Aristotelian and Parmenidean Logic', in *The Perennial Tradition of Neoplatonism*, ed. by J.J. Cleary, Leuven, Leuven University Press, 1997, p. 331-46, especially pp.337-38.

¹¹ J. Barnes, 'Logical Form and Logical Matter', in *Logica, mente e persona: Studi sulla filosofia antica*, ed. by A. Alberti, Firenze, Olschki, 1990, p. 79.

¹² Ibid., pp. 16ff.

¹³ Buzzetti, 'On Proclus' Comparison', p. 337.

¹⁴ W. Cavini, 'Chrisyppus on Speaking Truly and the Liar', in *Dialektiker und Stoiker: Zur Logik der Stoa und ihrer Vorläufer*, hrsg. von K. Döring und T. Ebert, Steiner, Stuttgart, 1993, p. 95.

served one' — a fact on which we shall have to come back later.¹⁵ A very similar idea lies behind John Stuart Mill's conception of a 'real inference' — an inference drawn directly from the minor premise of a syllogism to its conclusion, 'according to' the major premise taken as a rule.¹⁶ (One should notice that such a move is permitted precisely by interpreting a general proposition — the major premise — as a conditional assumption.) In more recent times, following Ramsey and Wittgenstein, the idea of an inference drawn *according to* a law-like statement — again a 'general' or 'open' hypothetical statement which provides a sort of 'inference-ticket', or 'warrant' — has been called up anew by Ryle and Toulmin.¹⁷ As has been pointed out by Ernest Nagel, the 'distinction between premises *from* which one reasons and rules *in accordance with* which inferences are drawn' is 'a sound one' and it is 'canonical in modern logical theory',¹⁸ but more to the point of the present discussion, in a brief note on Toulmin's inference theory, Otto Bird has conclusively shown that what the medieval logicians called a 'Topical Maxim' or a '*maxima propositio*' is in fact the 'traditional logic counterparts of Toulmin's Warrants'. In the medieval doctrine of the topics, a conditional maxim 'performs the same function as a warrant'¹⁹ and we find it actually described as 'a confirmatory rule that proves a consequence'.²⁰

So much, then, for the novelty of John of Glogovia's solution and the implausibility of its topical interpretation.

¹⁵ Buzzetti, 'On Proclus' Comparison', p. 338.

¹⁶ J. S. Mill, *A System of Logic Ratiocinative and Inductive (Collected Works, vols. 7-8)*, Toronto, University of Toronto Press-London, Routledge & Kegan Paul, 1974, p. 193.

¹⁷ Cf. G. Ryle, *The Concept of Mind*, London, Hutchinson, 1949; S. E. Toulmin, *The uses of argument*, Cambridge, Cambridge University Press, 1958; D. Buzzetti, 'La teoria dell'inferenza di John Stuart Mill e i logici di Cambridge', in *L'epistemologia di Cambridge: 1850-1950*, a cura di R. Simili, Bologna, il Mulino, 1987, pp. 101-119, and 'On Proclus' Comparison', pp. 339-40.

¹⁸ E. Nagel, Review of *The Philosophy of Science*, by Stephen Toulmin, in *Mind*, 63 (1954), p. 405.

¹⁹ O. Bird, 'The Re-discovery of the Topics: Professor Toulmin's Inference-Warrants', in *Mind*, 70 (1961), p. 537.

²⁰ Albertus de Saxonia, *Perutilis logica*, Nachdr. d. Ausg. Venedig 1522, Hildesheim, Olms, 1974, f. 33ra.

3. Basic misunderstandings of the topical solution, however, are not uncommon and the precise 'connection between topics and conditionals' —a connection medieval logicians were so keen on— has not always been carefully described. As a consequence, some historical reconstructions turn out to be somewhat distorted and important differences between opposing medieval doctrines, occurring *within* the topical tradition, are easily overlooked. The topical solution is based on a 'general thesis about conditionalization and deconditionalization',²¹ which may be recalled here as Sextus Empiricus once reported it:

They say that an argument is conclusive when the conditional which begins from the conjunction of its premises and ends in its conclusion is true.²²

This is a generally accepted principle: as for medieval authors, Christopher Martin has rephrased it in almost the same terms²³ and in modern textbooks too it reads very much alike.²⁴ But it is precisely the nature of this deduction 'metatheorem'²⁵ which is at stake, for its function does not only consist, as the assumption goes, in specifying what kind of formal requirements a conditional statement should meet in order to be true, but also in *distinguishing between the logical status of a premise and a rule*. And it should be kept in mind that the same sentence, taken either as a detached conditional or as an asserted premise, is respectively endowed with a different logical status and a different logical form. The very problem of the topical solution seems to consist precisely in defining what kind of logical relation holds between these

²¹ C. J. Martin, 'Something Amazing about the Peripatetic of Pallet: Abaelard's development of Boethius' account of conditional propositions', in *Argumentation*, 1 (1987), pp. 428 and 420.

²² Sextus Empiricus, *Adv. Math.* VIII 426 (Hülser, 1062). See also VIII 304, 415-18 (Hülser, 1059, 1060, 1065) and *Pyrr. Hypot.* II 137-38 (Hülser, 1058, 1064).

²³ Martin, 'Something Amazing', p. 420: 'An argument is good if and only if the conditional formed by taking the conjunction of its premisses as antecedent and its conclusion as consequent is true.'

²⁴ Cf. K. Lambert and B. C. Van Fraassen, *Derivation and Counterexample*, Encino, Calif., Dickenson, 1972, p. 12: 'An argument is valid if and only if the *conditional* statement, whose antecedent is the conjunction of the premises, and whose consequent is the conclusion of the argument, is true.'

²⁵ Martin, 'Something Amazing', p. 420.

two distinct forms. Serious theoretical shortcomings seem to ensue just from letting the one collapse onto the other.

It can plausibly be argued that what is implied in detaching a conditional premise from an argument and in assuming it as a rule for the resulting enthymeme is 'logical "ascent"'.²⁶ Gilbert Ryle has clearly pointed out that hypothetical statements expressing a rule 'belong to a different and more sophisticated level of discourse'²⁷ from that of the corresponding asserted premise. To quote but one example from late antiquity, Proclus' preferred use of 'hypothetical forms of argument'²⁸ as opposed to 'categorical syllogistic'²⁹ can indeed be seen 'as a sign of logical "ascent"'.³⁰ Ockham, as we shall see, performed exactly the opposite move, trying to avoid 'ascent' in hypothetical arguments. But present-day attempts at finding medieval forerunners for modern relevant³¹ and connexive logics³² simply overlook this point and end up blurring the distinction between opposing theories of material consequence.³³ The reason why Ivan Boh seems not to appreciate the viability of the topical solution appears to be due to his declared confidence in this perspective.³⁴

²⁶ For a fully expounded argument to this effect, see Buzzetti, 'On Proclus' Comparison', pp. 341-43.

²⁷ Ryle, *The Concept of Mind* (1949), Harmondsworth, Penguin Books, 1963, p. 118.

²⁸ Proclus, *In Parm.*, 1007.27.

²⁹ A. C. Lloyd, *The Anatomy of Neoplatonism*, Oxford, Clarendon Press, 1990, p. 16.

³⁰ Buzzetti, 'On Proclus' Comparison', p. 344.

³¹ Cf. Read, 'Formal and Material Consequence'. For relevant logic, see A. R. Anderson and N. D. Belnap, *Entailment: The logic of relevance and necessity*, 2 vols., Princeton, N.J., Princeton University Press, 1975-92 and S. Read, *Relevant Logic: A philosophical examination of inference*, Oxford, Blackwell, 1988.

³² C. J. Martin, 'Embarrassing Arguments and Surprising Conclusions in the Development of Theories of the Conditional in the Twelfth Century', in *Gilbert de Poitiers et ses contemporaines*, ed. by J. Jolivet et A. de Libera, Napoli, Bibliopolis, 1987, pp. 377-400; 'William's Machine', in *The Journal of Philosophy*, 83:10 (1986), pp. 564-72; and 'Something Amazing'. For connexive logic, see S. McCall, 'Connexive Implication', *The Journal of Symbolic Logic*, 31 (1966), pp. 415-33.

³³ Read, 'Formal and Material Consequence', pp. 238-41.

³⁴ For a 'very excellent account' of 'our present-century search for a viable version of a comprehensive relevant-logic', Boh refers us to Charles F. Kielkopf, *Formal*

A lack of clear insight, on the other hand, looks at least surprising when 'the higher-level status' of a 'maxim' as a 'rule' is clearly recognized, since one reads, for instance, that 'the words occurring in the maxims [...] belong to a second- or higher-level language',³⁵ but one reads, at the same time, that the connection of the topics with 12th-century theories of conditionals sounds 'rather unusual', or that 'it seems to have vanished during the last half of the century'.³⁶ As a matter of fact, however, the outcome of those discussions on the topics had a lasting influence more than two centuries afterwards and it was actually 'transmitted, especially by Peter of Spain, to late mediaeval logicians',³⁷ — a circumstance that is directly observable in Blasius' commentary to Peter's *Tractatus*. Thus, for a better understanding of Blasius' position, some insistence on the continuity of the topical tradition seems to be in order.

4. To Christopher Martin, Abelard's treatment of conditionals has offered a case for the connexive approach. Abelard's 'extensive and subtle account of conditional propositions' was however, in Martin's opinion, 'ultimately unsatisfactory'.³⁸ One wonders whether Abelard's endeavours towards 'a theory uniting topics and conditional propositions'³⁹ may not have been perceived as a major hindrance. As a matter of fact, the connexive approach is concerned about conditionals, not about the topics. It looks for necessary and sufficient conditions which define a true conditional and its primary interest lies in the import of its logical connective. From this point of view, it is the logical connective alone which is made responsible for the inferential force of an argument, whereas for Abelard the *vis inferentiae* consisted in the topical maxim taken as a whole. Moreover, the connexive approach mostly cares about how something follows *from* a conditional, and not about

Sentential Entailment, Washington, D.C., University Press of America, 1977 ('John of Glogovia's Rejection', p. 383).

³⁵ N. J. Green-Pedersen, 'The Topics in Medieval Logic', in *Argumentation*, 1 (1987), p. 412.

³⁶ *Ibid.*, p. 414.

³⁷ O. Bird, 'The Tradition of the Logical Topics: Aristotle to Ockham', in *Journal of the History of Ideas*, 23 (1962), p. 307.

³⁸ Martin, 'Something Amazing', p. 434.

³⁹ *Ibid.*, p. 426.

how something follows *according to* it. So it tends to overlook the difference between the logical status of a conditional rule and that of a conditional premise, whereas Abelard 'distinguished sharply between them', since the purpose of topical analysis was precisely to provide 'a metalogical explanation of the argument obtained by expanding [an] enthymeme'.⁴⁰ It seems, therefore, unlikely to do Abelard full justice in purely connexive terms.

One cannot, for instance, easily explain the presence of two apparently different 'accounts of the truth of conditionals' —the 'condition I' and the 'condition C', as Martin calls them.⁴¹ The first requires the 'inseparability' of the consequent: the antecedent cannot be true when the consequent is false;⁴² the latter requires the 'containment' of the consequent: the sense of the antecedent should contain the sense of the consequent.⁴³ From a relevantist point of view it is difficult to reconcile these principles. Condition I 'cannot meet' the connexive 'demands that Aristotle' —or Abelard for that matter— 'impose on conditionals'.⁴⁴ But if 'condition I infringes the connexive principles' Abelard 'cannot have an unqualified deduction theorem to connect conditionals and arguments'. If a conditional is true, it satisfies condition C, and the corresponding argument will satisfy condition I. But in general, 'entailment as expressed in true conditionals is not the converse of derivability or logical consequence as expressed in valid arguments'.⁴⁵ How could Abelard escape these strictures? One can surely wonder, but 'in the context of Abaelard's theories of arguments and conditionals', as Martin admits in the end, 'it is not amazing'⁴⁶ that he would have it both ways. But why?

⁴⁰ O. Bird, 'The Formalizing of the Topics in Medieval Logic', in *Notre Dame Journal of Formal Logic*, 1:4 (1960), p. 148.

⁴¹ C. J. Martin, 'William's Machine', p. 567.

⁴² Martin, 'Embarrassing Arguments', pp. 387-88. Cf. Peter Abelard, *Dialectica*, ed. by L.-M. de Rijk, Assen, Van Gorcum, 1956, p. 271: 'Id quod in antecedenti dicitur non potest esse absque eo quod in consequenti ponitur.'

⁴³ *Ibid.*, p. 392. Cf. Abelard, *Dialectica*, p. 284: 'Non solum antecedens absque consequenti non potest esse verum, (sed etiam) ex se ipsum exigit.'

⁴⁴ Martin, 'William's Machine', p. 567.

⁴⁵ *Ibid.*, p. 569.

⁴⁶ *Id.*, 'Something Amazing', p. 434.

There must be some point indeed for Abelard to express himself quite decidedly to that effect: 'although all necessary antecedents may be necessary *argumenta*, the converse does not occur'. For 'even though "Socrates is a man" necessarily argues that he is not a stone, it is not, however, necessarily antecedent to it'.⁴⁷ The necessity of the connection between *premises* and conclusion is not the necessity of the connection between antecedent and consequent of the conditional *rule*. The first depends on the force of the *propositional* connective of the conditional as a *premise*, the latter on the force of the *habitus* between the antecedent and the consequent of the same conditional taken as a *rule*. The logical status and the logical form of the conditional are quite different in the two cases. As a rule, a conditional maxim is a second-order statement. To represent it formally, we have to 'quantify over complex general terms' of the form ' $X(x)$ ',⁴⁸ i.e. over predicates. At careful inspection, all connexive constraints Abelard advances to restrict the application of propositional inferential rules, such as Conditional Simplification and Disjunctive Syllogism, appear to depend on the higher-order status of the maxims — a fact that relevantists should duly take into account. Suffice it to say, for instance, that 'the containment relation is guaranteed by the real definition' of a 'natural kind', such as man,⁴⁹ and it is well known that real definitions can be construed as second-order object-language statements. In brief, the conditions for the conditional maxim to be accepted as a rule are not the conditions for an inference from the corresponding premise to be valid. And a deduction theorem, just as well, defines a condition to justify validity, not a condition to justify the rule which justifies validity. Logicians have always striven after a formal justification of the validity requirements, but the necessity of an inference within a deductive system does not suffice to make that system a necessary one — and precisely to this point we shall have to come back again. The problem of checking for validity is not the problem of justifying deduction altogether.

⁴⁷ Pietro Abelardo, *Super Topica Glossae*, in *Scritti filosofici*, ed. by M. Dal Pra, Milano, Bocca, 1954, p. 309 (as translated by C. Martin, 'Something Amazing', p. 434).

⁴⁸ Martin, 'Something Amazing', p. 429.

⁴⁹ Id., 'William's Machine', p. 567.

Condition I for validity, as it has been said, does not satisfy connexive requirements for topical conditionals. But the 'insufficiency of condition I for the truth of a conditional' was rejected by the school of the Petit-Pont⁵⁰ and William of Soissons, one of Adam Balsham's students, invented a 'device(*machinam*)' for 'constructing unacceptable conclusions'⁵¹. Most probably 'the machine was a version of what we know as C. I. Lewis' proof that anything follows from a contradiction', and espousing Martin's reading it may be presented in this form:⁵²

- | | | |
|----|--|-----------------------------------|
| 1. | $A \ \& \ \neg A \rightarrow A$ | Conditional Simplification |
| 2. | $A \rightarrow A \vee B$ | Conditional Addition |
| 3. | $A \ \& \ \neg A \rightarrow A \vee B$ | 1, 2, Transitivity |
| 4. | $A \ \& \ \neg A \rightarrow \neg A$ | Conditional Simplification |
| 5. | $A \ \& \ \neg A \rightarrow (A \vee B) \ \& \ \neg A$ | 3, 4, Conditional Adjunction |
| 6. | $(A \vee B) \ \& \ \neg A \rightarrow B$ | Conditional Disjunctive Syllogism |
| 7. | $A \ \& \ \neg A \rightarrow B$ | 5, 6, Transitivity |

It is apparently unsafe to ground any historical reconstruction exclusively on the adoption or rejection of the two 'Parvipontanian principles' that 'anything follows from an impossibility and a necessity follows from anything'.⁵³ How can the *Nominales* be described 'followers of Abaelard',⁵⁴ if they 'shared with the Parvipontani the view that anything follows from the impossible', whereas 'Abelard maintained that anything did not follow from an impossibility'?⁵⁵ The affinity or divide between Abelard and the *Nominales* cannot probably be established leaving aside the discussions on the topics, for as far as the condition of inseparability is concerned, we have already seen that Abelard in fact could have taken it both ways. But what matters here is that Abelard

⁵⁰ Id., 'Embarrassing Arguments', p. 398.

⁵¹ John of Salisbury, *The Metalogicon*, transl. by D. D. McGarry, Berkeley, University of California Press, 1955, p. 98 (II.10).

⁵² Martin, 'William's Machine', p. 565.

⁵³ Ibid., p. 571.

⁵⁴ C. G. Normore, *The Tradition of Mediaeval Nominalism*, in J. F. Wippel (ed.), *Studies in Medieval Philosophy*, Washington, D.C., The Catholic University of America Press, 1987, p. 204.

⁵⁵ W. J. Courtenay, 'Nominales and Rules of Inference', in *Argumentationstheorie*, p. 158-59.

clearly 'grasped a purely formal element in the Topics' and was able to 'formulate it as a logical rule'. Even if

it is not yet possible—or it was not when Bird was writing—because of the lack of texts in 12th-13th century logic to show how this came about, [...] yet the 14th century treatises on Consequences have in effect incorporated his results.⁵⁶

Burleigh, for one, takes a maxim to be 'a rule through which a consequence holds'⁵⁷ and 'is willing to allow' that 'every Consequence is based on some dialectical Topic'.⁵⁸ As to Buridan, the *loci* are 'scarcely mentioned' throughout his work, but 'on the other hand certain ideas which used to be connected with the study of *loci* turn up as essential ingredients of his logic'.⁵⁹ Thus, by the 14th century, the topics have become 'absorbed into the theory of Consequences' and the pseudo-Scot 'explicitly identifies the Topics with one kind of Consequence',⁶⁰ the *consequentia materialis bona simpliciter*, which 'subdivides into many members according to the diversity of the dialectical topics'.⁶¹ And it is this kind of transformation, that sets the stage for Blasius' discussion.

Having so tried to vindicate the topical perspective, let us now turn to Blasius' *Questiones*. How can we evaluate Blasius' position from *within* the topical tradition?

5. Blasius Pelacani's treatment of the paradoxes of implication is to be found in his commentary on the *Tractatus*,⁶² 'the title by which

⁵⁶ Bird, 'The Formalizing of the Topic', p. 146.

⁵⁷ Walter Burleigh, *De Puritate Artis Logicae Tractatus Longior: With a revised edition of the Tractatus brevior*, ed. by P. Boehner, St. Bonaventure, N.Y., Franciscan Institute, 1955, p. 75.

⁵⁸ Bird, 'The Formalizing of the Topic', p. 148.

⁵⁹ S. Ebbesen, 'The Theory of *loci* in Antiquity and the Middle Ages', in *Argumentationstheorie*, p. 26.

⁶⁰ Bird, 'The Formalizing of the Topic', p. 145.

⁶¹ Ioannes Duns Scotus, *Opera omnia* (Reprograf. Nachdr. d. Ausg. Lyon, 1639), vol. 1, Hildesheim, G. Olms, 1968, p. 287-88: 'Consequentia materialis bona simpliciter [...] subdividitur in multa membra secundum diuersitatem locorum dialecticorum.'

⁶² The text of Blasius' *Questiones* or *Regule ac questiones super tractatus* is attested by two extant manuscripts, Venezia, Biblioteca Nazionale Marciana, lat. VI 63 (2550), ff. 1ra-92rb and Oxford, Bodleian, Canon. misc. 421, ff. 99ra-222vb, hereafter referred to, respectively, as *V* and *O*.

Peter of Spain's *Summulae logicales* were actually 'known'.⁶³ The study of this work, as part of the ordinary curriculum, was established by the statutes of the university of medicine and arts at Bologna in 1405,⁶⁴ but Alfonso Maierù has brought evidence to show that earlier commentaries on Tracts I-VI contained in MS 391 of the Biblioteca Antoniana of Padua 'corresponding to the entire program of readings codified in the statutes of 1405'⁶⁵ certify that 'these statutes represent the codification of longstanding teaching programs and practices, dating at least from the time when the university of medicine and arts was officially recognized in 1316'.⁶⁶ A careful analysis of the geographical references contained in the text suggests it as 'highly probable that Bologna was the place of origin of Blasius' questions'.⁶⁷ Among the other commentaries on the *Tractatus* of Peter of Spain written in Bologna during the 14th century, Blasius' *Questiones dialectice* are the only one composed in the form of questions. Some of them are referred to by the author as having been actually 'disputed',⁶⁸ and the work seems to 'have originated from the course given' to students in the years of Blasius permanence in Bologna,⁶⁹ where he taught logic at least in 1379-80.⁷⁰ The connection to actual teaching is emphasized by the insertion of a sophism in the first article of each question, sometimes unrelated to its

⁶³ A. Maierù, 'L'insegnamento della logica a Bologna nel secolo XIV e il manoscritto antoniano 391', in *Rapporti tra le università di Padova e Bologna*, a cura di L. Rossetti, Trieste, LINT, 1988 (Contributi alla storia dell'Università di Padova, 20), p. 6, transl. and rev. in Id., *University Training in Medieval Europe*, Leiden, Brill, 1994, p. 95.

⁶⁴ Cf. C. Malagola, *Statuti delle università e dei Collegi dello Studio bolognese*, Zanichelli, Bologna, 1888, p. 251.

⁶⁵ Maierù, *University Training*, p. 101.

⁶⁶ Ibid., p. 94 and Id., 'I commenti bolognesi ai *Tractatus* di Pietro Ispano', in *L'insegnamento della logica a Bologna nel XIV secolo*, a cura di D. Buzzetti, M. Ferriani e A. Tabarroni, Bologna, Istituto per la Storia dell'Università, 1992 (Studi e Memorie per la Storia dell'Università di Bologna, N.S., vol. VIII), pp. 497-500 and 502 in particular.

⁶⁷ Id., 'I commenti bolognesi', p. 505.

⁶⁸ Cf. *V*, ff. 46rb, 84rb, 84vb.

⁶⁹ Maierù, p. 516.

⁷⁰ Id., 'L'insegnamento', p. 2.

matter, but showing a 'precise plan of the master' clearly 'aimed at training students',⁷¹ on topics such as, for instance, obligations.⁷²

Blasius describes as 'general' the two questions on the paradoxes of implication, since the two 'propositions' they deal with —'every consequence whose antecedent is impossible is a sound one, whatsoever its consequent be' and 'every consequence whose consequent is necessary is a sound one, whatsoever its antecedent be'— can be assumed as logical rules: they 'are called rules by the logicians, as the logicians say that *the necessary follows from anything*, and also say that *from the impossible anything follows*'.⁷³ In accepting the two rules⁷⁴ Blasius joins a longstanding tradition, going back to the 'device' contrived by William of Sissons. He praises that argument as *modus deducendi specialissimus et excellentissimus* and expounds it in the following way:⁷⁵

- | | |
|---|---|
| 1. $A \ \& \ \neg A \rightarrow A$ | Conditional Simplification
(<i>ad partem antecedentis</i>) |
| 2. $A \rightarrow A \vee B$ | Conditional Addition
(<i>per principium in loyca</i>) |
| 3. $A \ \& \ \neg A \rightarrow \neg A$ | Conditional Simplification
(<i>ad partem antecedentis</i>) |
| 4. $\neg A \rightarrow \neg A$ | Reflexivity (<i>quia sunt eedem</i>) |
| 5. $A \ \& \ \neg A \rightarrow A \vee B$ | 1, 2, Transitivity (<i>a toto antecedenti</i>) |
| 6. $A \ \& \ \neg A \rightarrow \neg A$ | 3, 4, Transitivity (<i>a toto antecedenti</i>) |

⁷¹ Id., 'I commenti bolognesi', p. 514.

⁷² For a complete list of the sophisms, see Ibid., 'Appendice: I sofismi nelle *Questiones* di Biagio Pelacani', pp. 541-43.

⁷³ *V*, f. 75rb: 'Prima propositio est: quelibet (in qualibet *ms.*) consequentia cuius consequens est necessarium est bona consequentia quaecumque fuerit antecedens... Hec <erit> questio generalis et sequens determinationem istius. Secunda propositio: quelibet consequentia cuius antecedens est impossibile est bona consequentia quaecumque fuerit consequens. Et hec erit questio generalis quam paulo post disputabo. Et ille due propositiones posite dicuntur regule a loycis. Dicunt enim loyci quod *necessarium sequitur ad quodlibet*. Dicunt quoque <quod> *ad impossibile sequitur quodlibet*' (as transcribed by Maierù, 'I commenti bolognesi', p. 515).

⁷⁴ For Blasius' determinations of the two questions, see Appendix below.

⁷⁵ Cf. *O*, ff. 200va-vb; *V*, f. 77va (see Appendix below).

- | | |
|---|--|
| 7. $A \ \& \ \neg A \rightarrow (A \vee B) \ \& \ \neg A$ | 5, 6, Conditional Adjunction
(<i>ad totum consequens</i>) |
| 8. $(A \vee B) \ \& \ \neg A \rightarrow B$ | Conditional Disjunctive Syllogism
(<i>per principium</i>) |
| 9. $A \ \& \ \neg A \rightarrow B$ | 7, 8, Transitivity (<i>a primo ad ultimum</i>) |

The appraisal of Blasius position would amount to a trivial matter indeed, should we immediately assume as indicative of his notion of consequence 'the distinction made by C. J. Martin between the I(nseparability) and C(ontainment) account in his "William's Machine"'. But does that distinction really entail 'two different concepts of *consequentia*, each beginning a tradition of its own for the next five centuries'?⁷⁶ Should that be unquestionably the case, Blasius' appeal to William's 'most excellent' argument for the determination of Question 23 —his 6th question on Peter of Spain's Tract IV,⁷⁷ *Utrum quelibet consequentia cuius antecedens est impossibile sit bona*— would simply mean that he 'endorsed'⁷⁸ a non connexivist notion of consequence. But as we have seen, Abelard's contention that condition I, or 'the fact that it is impossible for the antecedent to be the case without the consequent' being the case, does not '*suffice* for the necessity of consecution, unless in addition to this the consequent be understood in the antecedent',⁷⁹ i.e. unless condition C holds as well, does not necessarily imply that we have to take it as the dividing point of two sharply opposed logical traditions. As a matter of fact, the two requirements are no more dissociable than a set of necessary conditions appears to be separable from a set of sufficient conditions for the proper foundation of a logical consequence. Rather, Abelard's contention takes us back to the basic question whether a 'necessary condition' for the 'intuitive understanding'

⁷⁶ I. Boh, *Epistemic Logic in the Later Middle Ages*, London, Routledge, 1993, p. 12.

⁷⁷ Cf. the list of questions given in G. Federici Vescovini, *Astrologia e scienza: La crisi dell'aristotelismo sul cadere del Trecento e Biagio Pelacani da Parma*, Vallecchi, Firenze, 1974, p. 417.

⁷⁸ Boh, *Epistemic Logic*. p. 12.

⁷⁹ Abelard, *Dialectica*, p. 407: 'Quae quidem omnes in eo necessarie videntur quod impossibile est esse antecedens absque consequenti. Nos autem id *sufficere* ad necessitatem consecutionis non concedimus, nisi etiam in antecedenti consequens, ut supra docuimus, intelligatur' (my emphasis).

of the notion of logical consequence 'is also a sufficient condition'.⁸⁰ Quite recently, John Etchemendy has argued this very point anew:

for an argument to be genuinely valid, it does not suffice for it to have a true conclusion or a false premise, for it simply to 'preserve the truth'. The truth of the premises must somehow *guarantee* the truth of the conclusion.⁸¹

It is 'this guarantee' —is it not significant that this expression should be so reminiscent of the 'inference warrant' or 'inference licence' jargon?— that gives rise to the 'familiar descriptions' of the consequence relation. And it is precisely to the medieval debate on 'the exact source of the perceived guarantee, whether it be the meanings of the expressions contained in the argument, brute logical intuition, or something else entirely',⁸² that we should now turn for a further attempt to clarify the matter.

6. Blasius' commentary on Peter of Spain unquestionably shows 'substantial traces of doctrines mostly of an English origin'.⁸³ As well as for his bipartition of supposition (as opposed to Ockham's tripartition),⁸⁴ Blasius' doctrine is very much indebted to Strode's, especially on conditional propositions. Blasius specifies necessary and sufficient conditions for the truth of a conditional in the following way: 'ad veritatem conditionalis requiritur et sufficit quod, si sit ita sicut significatur per antecedens, non aliter quin significatur per consequens denotetur esse',⁸⁵ and this *conclusio* follows almost literally Strode's definition of a sound consequence (*consequentia bona*): 'consequentia bona dicitur cuius non potest esse ita sicut adequate significatur per antece-

⁸⁰ John Etchemendy, *The Concept of Logical Consequence*, Cambridge, Mass., Harvard University Press, 1990, p. 77.

⁸¹ *Ibid.*

⁸² *Ibid.*, pp. 77-78.

⁸³ Maierù, 'I commenti bolognesi', p. 538.

⁸⁴ *Ibid.*

⁸⁵ *O*, f. 132va; *V*, f. 27va: 'For the truth of a conditional, it is both a necessary and sufficient condition, that if it is so as is signified by the antecedent, it shall not be denoted as being otherwise than is signified by the consequent.'

dens, quin sit ita sicut adequate significatur per suum consequens'.⁸⁶ As a matter of fact, Blasius refers to a conditional, whereas Strode refers to a consequence; but this is not surprising, since Blasius explicitly equates the truth of a conditional proposition to the soundness of a valid consequence: 'every true conditional is a sound consequence'.⁸⁷ It is apparently on the same grounds that Blasius feels justified to transfer the necessity of the consequence —the *necessitas consecutionis*, as Abelard would call it— to the conditional which grants its validity: 'every true conditional proposition is necessary'; and conversely, 'every false conditional is impossible'.⁸⁸ But is this a legitimate move? The import of such a strong commitment, brings us to the core of Blasius understanding of logical consequence. It is once again the relationship between a conditional assumed as a rule and the consequence granted by that same conditional which is here at stake. Are we justified in applying necessitation to a deduction theorem? Can we say that the rule according to which a consequent follows by necessity from an antecedent is a necessary rule? What is the criterion for the truth of a conditional and, accordingly, for the soundness of a consequence?

Indeed, the fallacy of necessitation seems to undermine the notion itself of logical consequence as commonly understood by modern logicians. John Etchemendy has detected it in Tarski's account of the concepts of logical truth and logical consequence⁸⁹ and it is worth recalling Etchemendy's refutation,⁹⁰ for medieval authors were well aware of that difficulty and able to provide appropriate answers. Tarski claims that his formal definition of logical consequence remains faithful to the intuitive, 'ordinary sense'⁹¹ from which we borrow its name, and that 'it

⁸⁶ Ralph Strode, *Consequentie*, Venice, 1491, f. 21vb: 'A sound consequence is said to be one which is such that it is impossible to be so as is adequately signified by the antecedent, without being so as is adequately signified by its consequent.'

⁸⁷ *O*, f. 132vb; *V*, f. 27vb: 'Omnis condicionalis vera est consequentia bona.'

⁸⁸ *Ibid.*: 'Omnis propositio condicionalis vera est necessaria...Omnis condicionalis falsa est impossibilis.'

⁸⁹ Cf. A. Tarski, 'Über den Begriff der Logischen Folgerung', in *Actes du Congrès international de philosophie scientifique*, Sorbonne, Paris, 1935, vol. 7: *Logique*, Paris, Hermann, 1936, transl. into English as 'On the Concept of Logical Consequence', in *Id.*, *Logic, Semantics, Metamathematics*, Oxford, Clarendon Press, 1956.

⁹⁰ Etchemendy, *The Concept of Logical Consequence*, pp. 85ff.

⁹¹ Tarski, 'On the Concept of Logical Consequence', p. 413.

can be proved, on the basis of this definition, that every consequence of true sentences *must* be true'.⁹² Now, Tarski's 'proof' that, if any sentence *S* in a first-order language is a Tarskian consequence of an arbitrary set *K* of such sentences, and if all of the members of *K* are true, then *S* must be true as well (and always so quite independently of our selected set *F* of logical constants), 'is perfectly correct'. However, this proof does not show 'that any *modal relation* holds between the premises and conclusion of the argument $\langle K, S \rangle$ ', for 'we would need to prove a theorem with an *embedded* modality'. Specifically, we would have to show that, for any *K* and *S*, if (1) '*S* is a Tarskian consequence of *K* (for some *F*)', then the following propositions, (2) 'All members of *K* are true', and (3) '*S* is false', are 'jointly incompatible'. But all we can show is 'the joint incompatibility of (1), (2), and (3)'. In conclusion,

we need only note the *fallaciousness* of any inference from 'Necessarily (if *P* and *Q* then not *R*)' to 'If *P* then necessarily (if *Q* then not *R*)'.

for it should be clear that 'the joint incompatibility of (1), (2), and (3), plus the truth of (1), does not entail the joint incompatibility of (2) and (3)'.⁹³

Since Blasius maintains, as we have seen, that 'every true conditional proposition is necessary',⁹⁴ we may remind that the fallacy applies also to Tarski's definition of logical truth. Also with logical truth there is 'an important modal feature' of our ordinary concept: 'a logical truth *must* be true —that is, it is *necessarily* true'. So it is important to show, for a definition of logical truth, that it satisfies its intuitive modal property. And again 'we can prove that if a sentence satisfies Tarski's definition of logical truth then it must be true', for 'after all, if it were not true, it would not satisfy the definition'. But,

unfortunately, this does not guarantee that the sentence has any peculiar modal properties, any more than the trivial observation 'if a sentence is true then it must be true' shows every truth to be a necessary truth.⁹⁵

⁹² Ibid., p. 417 (my emphasis).

⁹³ Etchemendy, *The Concept of Logical Consequence*, p. 87 (my emphasis).

⁹⁴ *O*, f. 132vb; *V*, f.27vb: see *supra*, note 88.

⁹⁵ Etchemendy, *The Concept of Logical Consequence*, p. 88.

So, is Blasius' contention liable to the same fallacy? As we have seen, he equates a true conditional to a sound consequence⁹⁶ and apparently understands it as the true logical rule which warrants the corresponding inference, but how does he justify its necessity? What kind of assurance does he provide, 'that arguments declared valid carry with them any independent guarantee of truth preservation, whether modal or epistemic or semantic'⁹⁷? On this point, again, Blasius is clearly dependant on Strode as, in his turn, Strode is on Ockham.

Both Strode and Ockham can certainly be ranked among the *loyci* Blasius appeals to and who take the two paradoxical principles as logical 'rules (*regulae*)'.⁹⁸ According to Strode, they are precisely the 'two rules' which apply to material consequence (*consequentia materialis*), the first being 'that from the impossible anything follows —i.e. every consequence whose antecedent is impossible is sound *de materia*'; and the second 'that the necessary follows from anything —i.e. every consequence whose consequent is necessary is sound *de materia*'.⁹⁹ But what is exactly, according to Strode, a sound material consequence? For 'one cannot expect that two different authors using the same terms of technical logical language will understand and apply these terms in the same way'.¹⁰⁰ He defines it in opposition to sound formal consequence (*consequentia bona et formalis*). A consequence sound *de forma* is described as a sound consequence —i.e. 'one which is such that if as is signified by the antecedent is understood, then so as is signified by the consequent is also understood'— in which 'the consequent is formally in the understanding of the antecedent'; for instance, the consequence 'If you are a man, you are an animal', complies with this description, for 'if anyone understands that you are a man, he understands that you are an animal'. On the other hand a consequence sound *de materia* is described as a sound consequence 'whose consequent is

⁹⁶ O, f. 132vb; V, f. 27vb: see *supra*, note 87.

⁹⁷ Etchemendy, *The Concept of Logical Consequence*, p. 94.

⁹⁸ V, f. 75rb: see *supra*, note 73.

⁹⁹ Ralph Strode, *Consequentie*, Venice, 1491, f. 21vb: 'Pro consequentia materialis sunt due regule: prima regula est quod ex impossibili sequitur quidlibet (quolibet, *ed.*), idest omnis consequentia cuius antecedens est impossibile est bona de materia; secunda regula est quod necessarium sequitur ad quidlibet (quodlibet, *ed.*), idest omnis consequentia cuius consequens est necessarium est bona de materia.'

¹⁰⁰ K. Jacobi, 'Introduction II', in *Argumentationstheorie*, p. 118.

not formally in the understanding of the antecedent'.¹⁰¹ Strode goes on to say that 'from these descriptions it appears, that every consequence which is sound and formal (*consequentia bona et formalis*) is also sound and material, but not vice versa'.¹⁰² Now, a material consequence is clearly one which satisfies a truth-functional requirement, such as condition I. But this requirement, which is certainly necessary for a formal consequence to be sound, is by no means a sufficient condition. A sound formal consequence must satisfy a containment condition, which 'is clearly "epistemic" rather than "alethic"',¹⁰³ according to the way Strode describes it. So, by the look of it, Abelard's condition C 'that from the sense of the antecedent the sense of the consequent is required'¹⁰⁴ has turned into an epistemic definition of consequence. Is, in Strode's opinion, the ultimate justification of its necessity of this kind?

7. A similar orientation towards an epistemic solution is to be found in Ockham. More openly than Strode—who implies it by maintaining that 'just as' we can say that something holds for a 'conditional', in a quite 'similar way' we can say that the same holds for a 'consequence', which 'commonly' is 'called a conditional proposition'¹⁰⁵—Ockham himself supports Blasius' equation of consequences and conditionals, by stating directly that 'a conditional is equivalent to a consequence'.¹⁰⁶ And just like Strode, Ockham assumes the two paradoxical principles as rules for material consequence and so proper to it, that

¹⁰¹ Strode, *Consequentie*, f. 21vb: 'Consequentia bona de forma dicitur esse cuius, si sic esse sicut adequate significatur per antecedens intelligitur, sic esse sicut adequate significatur per consequens intelligitur, ut 'tu es homo, ergo tu es animal.' Si quis enim intelligat te esse hominem, intelligit te esse animal; et ideo dicitur, in tali consequentia, consequens esse de formali intellectu antecedentis. Sed consequentia bona materialis tantum dicitur cuius consequens non est de formali intellectu antecedentis, servatis tamen conditionibus requisitis ad consequentiam bonam, ut supra dictum est.'

¹⁰² Ibid.: 'Ideo patet ex dictis descriptionibus, quod omnis consequentia bona et formalis est bona et materialis, sed non e converso.'

¹⁰³ Boh, *Epistemic Logic*, p. 91.

¹⁰⁴ Abelard, *Dialectica*, p. 253.

¹⁰⁵ Strode, *Consequentie*, f. 21vb: 'Sicut ergo dicebatur... conditionalem esse... similiter dicitur de qualibet consequentia que nuncupatur propositio conditionalis (rationalis, ed.)... communiter.'

¹⁰⁶ William of Ockham, *Summa logicae*, II, cap. 31.

apparently 'for him there are no other cases of material consequence'.¹⁰⁷ To our purpose, it is worth considering how he comes to that, by way of radical 're-thinking and re-interpreting the traditional doctrine of the topic'.¹⁰⁸ With a point of view completely of his own Ockham still moves, more than is usually acknowledged,¹⁰⁹ within the topical tradition. Although 'he not only changes or drops many of the traditional Topical distinctions, but he also abandons the very name of Topics as designating a distinct division of logic', by overlooking the content of that tradition, one 'might well miss the fact that most of his treatise on Consequences in his great logic is a re-working of the Topics'.¹¹⁰ And it is now interesting to see 'how Ockham recast the topical tradition'.¹¹¹ What kind of solution did he contrive?

Ockham's topical solution is characterised by the elimination of logical 'ascent'.¹¹² We 'approximate more closely' to the way Ockham reformulates topical rules, if in a formal rendition, as Bird expounds, we 'drop' all predicate quantifiers.¹¹³ And it can be shown that this amounts to *abolishing the distinction between the logical status of a premise and a rule*. We may consider, for instance, the two paradoxical 'rules', which are 'the extrinsic medium (*medium extrinsecum*) because of which a material consequence holds'.¹¹⁴ Now, it is precisely 'when we consider what validates the passage from antecedent to consequent', that 'we find a topical consideration', and that implies, 'in Ockham's language', showing 'through what medium a consequence

¹⁰⁷ F. Schupp, 'Zur Textrekonstruktion der formalen und materialen Folgerung in der kritischen Ockham-Ausgabe', in *Argumentationstheorie*, p. 216; cf. Ockham, *Summa logicae*, III-3, capp. 1 and 38.

¹⁰⁸ O. Bird, 'Topic and Consequence in Ockham's Logic', in *Notre Dame Journal of Formal Logic*, 2:2 (1961), p. 74.

¹⁰⁹ Cf. *Ibid.*, p. 65: 'Boehner was at least over-hasty and inaccurate in saying that Ockham has only a "loose arrangement" and "wisely omitted" a systematic ordering of the Topical consequences. Furthermore, previous study of Ockham's theory of Consequences – Salamucha, Boehner, Moody – has paid practically no attention to those Consequences which derive immediately from Topical analysis' (reference is made to P. Boehner, *Medieval Logic*, Manchester University Press, 1952, p. 55).

¹¹⁰ Bird, 'The Tradition of the Logical Topics', p. 317.

¹¹¹ *Id.*, 'Topic and Consequence in Ockham's Logic', p. 65.

¹¹² Cf. *supra*, note 26.

¹¹³ Bird, 'The Tradition of the Logical Topics', p. 319.

¹¹⁴ F. Schupp, 'Zur Textrekonstruktion', p. 217.

holds'.¹¹⁵ As we have seen, in the third part of his *Summa logicae*, he treats conditionals as consequences, but he drops the traditional language of the topics (topical 'maxim' and topical 'difference') and refers to an 'extrinsic' or to an 'intrinsic medium' instead.¹¹⁶ But 'the different kinds of consequence cannot be immediately established through the distinction between *medium extrinsecum* and *medium intrinsecum*'. As has been shown most appropriately by Franz Schupp, who has rectified the textual reconstruction of the Ockham edition, Ockham cares about grounding all sorts of consequence on an extrinsic medium. However, whereas a material consequence 'can thus be connected in a circuitous way to a truth-functional property of the inference', this cannot always be the case for a formal consequence. More precisely, 'formal and material consequences distinguish themselves through the fact, that a material consequence holds because of an extrinsic medium, which is connected to the *generales condiciones propositionum* (truth, falsity, necessity, impossibility), whereas this is *not* the case for a formal consequence'.¹¹⁷ Thus, we have two kinds of formal consequence, 'some of them hold by virtue of an extrinsic medium, which respects the form of the proposition',¹¹⁸ i.e. because of 'rules which allow the *syntactical* transformation of a proposition',¹¹⁹ but 'some of them hold immediately through an intrinsic medium and mediately through an extrinsic medium which <does not> respect the general conditions of propositions, such as truth or falsity, necessity or impossibility'.¹²⁰ What this

¹¹⁵ Bird, 'The Tradition of the Logical Topics', p. 318.

¹¹⁶ Ockham, *Summa logicae*, III-3, cap. 1: 'Consequentia tenet per medium intrinsecum quando tenet virtute alicuius propositionis formatae ex eiusdem terminis... Consequentia autem quae tenet per medium extrinsecum est quando tenet per aliquam regulam generalem quae non plus respicit illos terminos quam alios.'

¹¹⁷ F. Schupp, 'Zur Textrekonstruktion', p. 216.

¹¹⁸ Ockham, *Summa logicae*, III-3, cap. 1: 'Consequentia formalis est duplex, quia quaedam tenet per medium extrinsecum, quod respicit formam propositionis.'

¹¹⁹ F. Schupp, 'Zur Textrekonstruktion', p. 217.

¹²⁰ Ockham, *Summa logicae*, III-3, cap. 1. The text quoted here follows Schupp's reconstruction ('Zur Textrekonstruktion', p. 215), i.e. 'Quaedam tenet per medium intrinsecum immediate et mediate per medium extrinsecum <non> respiciens generales condiciones propositionum ut veritatem vel falsitatem, necessitatem vel impossibilitatem', as opposed to the patently incorrect text of the edition, i.e. 'Quaedam tenet per medium intrinsecum immediate et mediate per medium extrinsecum

means is that in the latter case 'the consequence holds because of an extrinsic medium dependent on a *semantic* relation between antecedent and consequent, which is expressed by the intrinsic medium'. So, for instance, the consequence 'Sortes currit, ergo homo non currit', holds only indirectly through the extrinsic medium *a singulari ad indefinitam postposita negatione est bona consequentia*, but holds immediately through the intrinsic medium 'Sortes est homo'. And it is just 'at this point' that 'the theory of *loci* becomes relevant'.¹²¹

But here we should duly keep in mind Ockham's particular use of *loci* in his deduction of an extrinsic medium. Without entering into a detailed analysis,¹²² we may just recall, for instance, that when he expounds the function of an intrinsic medium such as a true general proposition—a topical difference assumed as a warrant of an enthymematic inference—the crucial shifting from a higher-order logical status (such as that of the topical principle) to a first-order logical status (such as that of the corresponding asserted premise) becomes 'apparent in his using the superior-inferior relation rather than that of genus-species'. This decision enables him to 'drop the universal predicate quantifier', and 'to consolidate under one rule two of Peter of Spain's Maxims'. But in doing so 'Ockham has obviously moved further than [any] previous author towards an extensional point of view',¹²³ and 're-interpreted the traditional Topical analysis'.¹²⁴ Thus,

what he calls the rule (*regula*) expressed in the extrinsic medium obviously corresponds to the Topical Maxim. But from this it does not follow that the intrinsic medium corresponds in every respect to the topical difference.

It obviously does 'to some extent', but 'the expression of the intrinsic medium amounts to the assertion of the antecedent of the extrinsic medium' where terms are taken 'in personal and significative supposition' as opposed to 'simple supposition': a proposition of the form '*A* is *B*' is interpreted as a first-order propositional function of the form

respicens generales condiciones propositionum, non veritatem vel falsitatem, nec necessitatem vel impossibilitatem.' The difference is substantial!

¹²¹ F. Schupp, 'Zur Textrekonstruktion', p. 217.

¹²² For an accurate account, see Bird, 'The Tradition of the Logical Topics'.

¹²³ *Ibid.*, p. 317.

¹²⁴ *Id.*, 'Topic and Consequence in Ockham's Logic', p. 73.

' $(x) (x \in A \rightarrow Bx)$ ' and not as a second-order *de re* statement about real kinds. The extrinsic medium which grants the inference 'is now on a different semantic level' from that of the topical principle. Instead of a higher-order rule, now we get a 'logical law',¹²⁵ a theorem using referential variables, or else what is appropriately described as an 'inference-scheme'¹²⁶ using metalinguistic variables.

Now, what does the elimination of logical 'ascent' actually imply? We may try to give an answer by returning to Tarski's notion of logical consequence. According to Etchemendy, Tarski 'was well aware that his notion of *logical* consequence reduces to *material* consequence — that is mere truth preservation— when all expressions are included in F , i.e. the set of logical constants'.¹²⁷ And this is exactly what Ockham does, by including the intrinsic in the extrinsic medium — a logical law — as its asserted antecedent. The conclusion, as Tarski himself points out, 'would in this case follow from the class' of premises 'if either [the conclusion] were true or at least one sentence of the class [of premises] were false'.¹²⁸ The paradoxes of *material* implication would then become paradoxes of *logical* implication and the import of a metalinguistic notion, such as that of logical consequence, would be transferred onto the meaning of a logical constant, such as the connective of material implication. (Connexive and relevant logics do not seem to fare very well either by ignoring, just as well, logical 'ascent'.) This seems very likely to be the reason why Blasius, following Ockham and Strode, openly falls prey to the fallacy of necessitation: *omnis condicionalis vera est consequentia bona* and *omnis propositio condicionalis vera est necessaria*.

8. As a matter of fact, Ockham admits of rules for 'a distinct kind of consequences', where 'the predicate of the consequent is the name of one of the predicables and thereby puts its subject-term in simple supposition'.¹²⁹ And the recognition of higher-order rules comes about also

¹²⁵ Id., 'The Tradition of the Logical Topics', pp. 319-20.

¹²⁶ Id., 'The Formalizing of the Topic', p. 142.

¹²⁷ Etchemendy, *The Concept of Logical Consequence*, p. 92.

¹²⁸ Tarski, 'On the concept of Logical Consequence', p. 419.

¹²⁹ Bird, 'The Tradition of the Logical Topics', p. 321. Cf. Ockham, *Summa logicae*, III-3, capp. 1 and 18.

in his theory of 'demonstrative syllogism'.¹³⁰ As to the kinds of propositions involved in a strict, scientific demonstration, Ockham has to say:

Of propositions required for a demonstration some are parts of demonstration, such as two premises and one conclusion, while others are not parts of demonstration and are called dignities or *maxims* or suppositions; these do not enter into a demonstration in their own proper form, yet it is by virtue of them that the premises of demonstration are somehow known.¹³¹

Here we find a clear distinction between asserted premises and maxims by means of which they are known. As an example, Ockham gives 'every heat is productive of heat' and 'everything hot is productive of heat' and insists that they are distinct from one another; then he adds that

the former is prior and the latter posterior, but nevertheless the latter cannot be properly demonstrated through the former; moreover, the second one enters into demonstrations, while the first one does not, and yet many demonstrations hold in virtue of the first one which cannot enter into them.¹³²

No clearer statement of the different status of the premises and the maxims could be provided. But is it a difference of *logical* status? Do we have here a higher-order proposition in simple supposition as opposed to a first-order one in personal supposition? It very much depends on the interpretation of Ockham's semantic views.¹³³ But here

¹³⁰ Ibid., III-1, cap. 1: 'Syllogismus demonstrativus est ille in quo ex propositionibus necessariis evidenter notis potest adquiri prima notitia conclusionis.'

¹³¹ Ibid., III-2, cap. 4: 'Propositum requisitarum ad demonstrationem quaedam sunt partes demonstrationis, sicut duae premissae et una conclusio, et quaedam non sunt partes demonstrationis. Et vocantur dignitates et *maximae* vel suppositiones, quae sub propria forma non ingrediuntur demonstrationem, virtute tamen illarum propositionum aliquo modo sciuntur praemissae demonstrationis.'

¹³² Ibid., III-2, cap. 4: 'Istae igitur sunt distinctae propositiones "omnis calor est calefactivus"... "omne calidum est calefactivum"; et prima prior est et secunda posterior. Et tamen secunda non potest proprie demonstrari per primam et secunda intrat demonstrationem non prima, et tamen virtute primae tenent multae demonstrationes quas non potest ingredi.'

¹³³ For a higher-order interpretation of propositions of this kind in late 14th-century Bolognese natural philosophy, see D. Buzzetti, 'Lo strano caso dell'*intensio* e la sto-

the ultimate ground for priority seems to be rather an 'epistemic'¹³⁴ one: the first proposition comes 'prior' in the order of knowledge, as it is 'distinctly known through experience',¹³⁵ so that through the second, which is dependent on it and 'posterior' in the order of knowledge, the 'evident cognition' of a necessary conclusion, of 'something true and necessary', can be 'caused' as by one of 'the premises used to support it in a discursive syllogism'.¹³⁶

Even when he makes allowance for a distinction between premises and maxims, it is not clear whether Ockham admits at all logical 'ascent'. Certainly he does not in his theory of formal and material consequence as opposed to pseudo-Scotus'. According to the latter, a formal consequence is 'one that holds for all terms when there is a similar disposition and form of the terms', whereas a material consequence is 'one which does not hold for all terms retaining a similar disposition and form' but with a variation of the terms.¹³⁷ So, what pseudo-Scotus calls material consequence is indeed to be compared not with Ockham's material, but with Ockham's formal consequence *per medium intrinsicum*. It cannot be said, then, that they both 'subscribed to' the same 'general picture'.¹³⁸ The difference, though, is not just a matter of classifying consequences. Their different categorization can explain why every material consequence, in pseudo-Scotus' sense, can be reduced to a formal one, but not vice versa, as well as why every formal consequence, in Ockham's (and Strode's) sense, is also a material one, but not vice versa. However, all that does not explain, beyond terminology,

ria della logica medievale', in *Rivista di storia della filosofia*, N.S., 51:1 (1996), pp. 95-111.

¹³⁴ Boh, *Epistemic Logic*. p.

¹³⁵ Ockham, *Summa logicae*: 'Verbi gratia, ista est una propositio prima, per experientiam praecise nota "omnis calor est calefactivus", quae non potest esse pars demonstrationis.'

¹³⁶ Id., *Ordinatio*, Prol., q. 2, art. 1: 'Scientia... est notitia evidens veri necessarii, nata causari per praemissas applicatas ad ipsum per discursum syllogisticum.'

¹³⁷ Pseudo-Scotus, *In librum primum Priorum Analyticorum Aristotelis quaestiones*, in *Johannis Duns Scoti Opera Omnia*, ed. L. Wadding (Lyon, Durand, 1639), repr. Hildesheim, Olms, 1968, vol. 1, p. 287b: 'Consequentia formalis est illa, quae tenet in omnibus terminis, stante consimili dispositione, et forma terminorum... Sed consequentia materialis est illa, quae non tenet in omnibus terminis, retenta consimili dispositione, et forma, ita quod non fiat variatio nisi terminorum.'

¹³⁸ Read, 'Formal and Material Consequence', p. 240; cf. *supra*, note 4.

the real difference. A clear insight, in this respect, can only be gained from *within* the topical tradition. It is the *logical* status of an intrinsic medium, as opposed to a topical difference, which matters here. Ockham's intrinsic medium, as we have seen, is the asserted antecedent of a first-order logical law, whereas pseudo-Scotus' topical difference is a higher-order object-language *de re* statement about 'formalities' —the Scotist notion of a formal distinction *a parte rei* comes here obviously into play. Ockham's truth-functional and epistemic solutions, then, are not the only viable medieval answer to the problem of logical consequence. Logical 'ascent', as preserved in the Scotistic notion of material consequence, seems to provide a reliable alternative. But how does Duns Scotus himself respond to it?

9. Scotus' answer to our problem comes precisely from a refutation of the modal fallacy to which Tarski seems, by all appearances, to have fallen prey'.¹³⁹ Now, the 'key' to that fallacy 'is the ambiguous scope of the modality in question'¹⁴⁰ and it is precisely this ambiguity that Scotus points out, distinguishing the two senses of a complex proposition such as 'everything which exists, when it exists, is necessary', or 'that an animal is running, if a man is running, is necessary'.¹⁴¹ Such propositions can be taken in a composite (*sensus compositionis*) or in a divided sense (*sensus divisionis*) and their import is respectively different, as to what they say to be necessary. And if what can be taken in a composite or in a divided sense is a conditional, necessity can be said to apply either to the consequence it expresses (*necessitas consequentiae*), or to its consequent (*necessitas consequentis*). But the point here is a subtle one: in the first case (*in sensu compositionis*) the

¹³⁹ Etchemendy, *The Concept of Logical Consequence*, p. 92.

¹⁴⁰ *Ibid.*, p. 90.

¹⁴¹ John Duns Scotus, *Lectura*, I, d. 39, qq. 1-5, n. 58 (ed. Vat., XVII, p. 499): 'Ad primum dicendum quod haec "omne quod est, quando est, est necessarium", est distinguenda secundum compositionem et divisionem, — sicut haec "animal currere, si homo currit, est necessarium". Et in sensu compositionis est categorica et vera, et tunc denotat necessitatem consequentiae, et est sensus 'animal currere si homo currit, est necessarium', hoc est: haec est necessaria 'animal currit si homo currit.' Sensus divisionis est 'animal currere, si homo currit, est necessarium:' est tunc hypothetica et falsa, et est sensus 'animal currere est necessarium, si homo currit', et tunc denotat necessitatem consequentis.

conditional is expressed by a categorical proposition (*est categorica*), whereas in the latter (*in sensu divisionis*) it is expressed by a hypothetical proposition (*est hypothetica*). Let us consider Scotus' example, 'that an animal is running, if a man is running, is necessary'.

In the composite sense, the conditional 'an animal is running if a man is running' (*animal currit si homo currit*) expresses a categorical proposition, and it means 'a running man is a running animal', so that its necessity denotes the necessity of the consequence 'if something is a running man, then it is a running animal'. Now, this consequence can only be necessary if we assume the proposition 'all men are animals' as a rule (a topical difference), because it is an enthymeme which can be completed only by the assumption of a major premise corresponding to that rule, namely 'every man is an animal' or 'if something is a man, then it is an animal'. Since the rule is true, as an instance of a topical difference, the conditional is necessarily true. Putting it all together: in the composite sense the conditional expresses a categorical proposition and is true (*categorica et vera*); the true conditional is seen as a necessary consequence; and the necessary consequence implies the assumption of a higher-order rule. This rule, however, can be said to be necessary, or not necessary, on entirely different grounds. But if we take the conditional in the divided sense, it is quite another matter. In this case the conditional 'an animal is running, if a man is running' (*animal currit, si homo currit*) expresses a hypothetical proposition, and it means 'that an animal is running is necessary, if a man is running'. So its necessity denotes the necessity of the consequent, which can also be expressed by a conditional such as 'if something is an animal, then it is running'. But in order to be necessary, this last conditional would require the assumption of 'all animals are running' as a higher-order rule, which is obviously false (*tunc hypothetica et falsa*).

This is the kind of reconstruction Scotus' argument admits of inside the topical tradition. And it looks very plausible if we acknowledge, as a basic assumption of most medieval logicians, what Ernest Moody has described in the following way:

it was assumed, in mediaeval logic, that adequate laws governing the use of language could, without contradiction or paradox, be developed within language. It was the task of logic to develop these laws,

to be applied not only in the positive sciences and in philosophy but in logic itself.¹⁴²

As a matter of fact, object-language higher-order statements are one of the 'several procedures adopted by the medievals' to provide 'an account of the meaning of names'.¹⁴³ Such statements can be parsed as made up by a 'higher order' copula, which 'takes as arguments' expressions of a functorial, or predicative kind.¹⁴⁴ Now, higher-order *de re* statements 'based on an "...is..." which forms a proposition out of two *functors* (instead of two names)' can be considered equivalent to 'a *de voce* account of the meaning of a certain word',¹⁴⁵ that is to say a metalinguistic statement which defines its proper use. Both are 'alternative and equally acceptable versions', or 'formulations', of 'the talk about meaning'¹⁴⁶ and medieval authors were quite ready to exploit this 'equivalence between second-order *de re*, or object-language statements' on the one hand and 'metalinguistic statements, or statements on intentions (*de intentionibus*), about the meaning of expressions used in first-order statements'¹⁴⁷ on the other. In the topical tradition, such object-language higher-order discourse —*quidditative*¹⁴⁸ discourse— is used to account for the necessity of conditionals, or for the soundness of consequences, by describing the relation between the antecedent and the consequent in terms of a higher-order rule or topical principle. In this way, both the syntactical and truth-functional aspect of an inference, such as expressed by the syntactical form of propositions and arguments, and the semantic aspect of relevant inference or material consequence (in pseudo-Scotus' sense), such as contained in the ordinary notion of logical consequence, can be accounted for by means of a

¹⁴² E. A. Moody, *Truth and Consequence in Mediaeval Logic* (1953), Westport, Conn., Greenwood Press, 1976, p. 110.

¹⁴³ D. P. Henry, *That Most Subtle Question* (Quaestio subtilissima): *The metaphysical bearing of medieval and contemporary linguistic disciplines*, Manchester, Manchester University Press, 1984, p. 92.

¹⁴⁴ Id., *Medieval Logic and Metaphysics: A modern introduction*, London, Hutchinson, 1972, p. 43.

¹⁴⁵ Id., *That Most Subtle Question*, p. 93.

¹⁴⁶ Id., *Commentary on De grammatico: The historical-logical dimensions of a dialogue of St. Anselm's*, Dordrecht, Reidel, 1974, p. 32.

¹⁴⁷ Buzzetti, 'Lo strano caso dell'*intensio*', p. 101.

¹⁴⁸ Cf. Henry, *That Most Subtle Question*, *passim*.

linguistic self-reflecting device. Rules are not provided in a metalinguistic form, as inference-schemes containing metalinguistic variables, nor in the form of theorems containing referential variables, but as higher-order object-language statements. The justification of inference can be transformed into a linguistic feature without having to pay the price of regarding 'all terms of the language as logical'.¹⁴⁹ Language is endowed with a self-reflecting feature and enabled to express 'its own theory of meaning'.¹⁵⁰ And that much can be achieved without falling into the modal fallacy of necessitation and without reducing the notion of logical consequence to the purely truth-functional notion of modern material implication.

10. We can now try to sum up our long, extended argument aimed at clarifying the role of 'quidditative' discourse in most medieval attempts to justify logical consequence within the topical tradition. This kind of medieval solution does not find any precise modern counterpart and may therefore turn out to be particularly suggestive of promising new approaches. As we have seen, it may be regarded as an attempt at transforming into a linguistic feature — a built-in and self-reflexive character of the object-language — the epistemological justification of a relevant inference.¹⁵¹ In this respect, it exemplifies one of the two commonly practised alternative forms of linguistic stipulation. In fact, a process aimed at bridging the gap between syntax and semantics can be carried out either through the introduction of (i) *de voce* nominal definitions expressed by appropriate meta-linguistic statements, or (ii) *de re* object-language definitions construed as statements endowed with a special linguistic and logical status. In the medieval tradition, the first option was mostly chosen by such authors as Ockham and Buridan, whereas the second was preferred by Scotus and the kind of nominalists

¹⁴⁹ Tarski, 'On the concept of Logical Consequence', p. 419.

¹⁵⁰ A. N. Prior, 'Some Problems of Self-reference in John Buridan', in *Proceedings of the British Academy*, 48 (1962), p. 292.

¹⁵¹ In modern terms the problem emerges in discussions connected with the kantian notion of *a priori* synthetic judgements, or with Carnap's idea of 'meaning postulates' (cf. R. Carnap, *Meaning and Necessity: A study in semantics and modal logic*, Chicago, University of Chicago Press, 1947).

who were ready to admit *dicta* or *complexe significabilia*.¹⁵² The choice between the two different options very often depends on opposing views in the philosophy of logic as to the theory of meaning and the structure of the object-language. The first option is usually associated with referential theories of meaning, the latter with the admission of semantic notions —such as Anselm's *significatio per se*, Abelard's *status* and *dicta*, and Gregory of Rimini's *complexe significabile*— not reducible to actual instances of 'spoken or inscribed' expressions, of 'mental events',¹⁵³ or external things to which they may refer. In other words, according to the latter point of view, the realm of semantic notions is to be kept apart from the respective realms of uttered words and sentences, of their mental equivalents, and external non-linguistic entities. The other aspect in which this opposition shows itself emerges in terms of language hierarchy. Quite appropriately, an all too direct transposition of the modern object-language/meta-language distinction into medieval terms can be objected to.¹⁵⁴ Anachronism would by all means be unavoidable. But we could just as well express the same opposition in terms of the *de voce/de re* distinction. And what matters here is not so much hierarchy as such, but the way in which it is established. Now, according to the first option, we can speak about an object-language only through first-order *de voce* statements, whereas according to the second, we allow a language to be self-reflexive by means of second-order *de re* statements. It is only through the latter option, that a justification of logical inference can be afforded, which avoids the snares of paradoxical implications, or the modal fallacy that trivializes the very notion of logical consequence. Scotus' solution is a solution of this kind —a linguistic solution which is able to keep track of the contingent origin of the principle of a material implication, the *differentia maximae* of the topical tradition. It was precisely by comprising a self-reflexive feature in the capabilities of the object-language, that Scotus was able to dispose of the fallacy of necessitation. In such a way, the justification of deduction gets connected to the very working

¹⁵² Cf. Normore, *The Tradition of Mediaeval Nominalism*, especially pp. 207-09 and 212-15.

¹⁵³ Cf. Martin, 'Something Amazing', p. 426.

¹⁵⁴ Cf. van der Lecq, 'The Role of Language-levels.'

of the object-language and relevant inferences can be accounted for by means of a suitable formalization of inbuilt linguistic features.¹⁵⁵

The implications of Scotus' position are indeed far reaching, especially to avoid any modern mishandling of his notion of possible worlds. The conviction that possible-world semantics may have originated in Scotus' ideas seems almost to have turned into a widespread historiographic myth. How much his outlook could induce us to diverge from a standard model-theoretic approach might already be glimpsed from what has just been said. But pursuing this discussion would take us far beyond the scope of the present essay.

¹⁵⁵ Cf. Bird, 'The Tradition of the Logical Topics', p. 323.

APPENDIX

Blasii Pelacani *Quaestiones Dialecticae*

O = Canonicianus (Oxford, Bodleian Library, Canon. misc., 421, ff. 199vb-201ra; 202vb-202rb)

V = Marcianus (Venezia, Biblioteca Nazionale Marciana, lat. VI 63 (2550), ff. 77rb-78ra; 78vb-79ra)

< > : supplevi

... : usque ad (inclusive)

(!) : sic

add. : addit

om. : omittit

< In Tract. IV, Quaest. 6, Art. 2 >

Vigesima tertia questio:¹⁵⁶ *Utrum quelibet consequentia cuius antecedens est impossibile sit bona.*

[...] Pro secundo articulo, [*O200ra*] noto quod ista questio est mota¹⁵⁷ principaliter ut nobis pateat¹⁵⁸ modus deducendi consequentiam qua arguitur, vel qua dicitur, quod ad impossibile¹⁵⁹ sequitur¹⁶⁰ quidlibet, quia hoc viso videbitur determinatio questionis. [*V77va*]

Et ideo scribit una opinio, quod ad impossibile sequitur quidlibet. Isto modo¹⁶¹ presupponit,¹⁶² primo, quod omnis illa consequentia est bona cuius impossibile est antecedens esse verum sine consequente et ex isto dicit quod ad impossibile sequitur quidlibet. Quod patet, quia quacumque consequentia

¹⁵⁶ Vigesima...questio]*V om. O*

¹⁵⁷ est mota]*O om. V*

¹⁵⁸ nobis pateat]*V pateat nobis O*

¹⁵⁹ impossibile]*V possibile O*

¹⁶⁰ sequitur]*V sequatur O*

¹⁶¹ modo]*V om. O*

¹⁶² presupponit]*O presupponit V*

data, cuius antecedens est impossibile, illius consequentie antecedens non potest esse verum sine consequente.

Quod probat¹⁶³ sic: quia si non, stabit oppositum, scilicet talis consequentie possibile est antecedens esse verum sine consequente; tunc arguitur sic: talis consequentie possibile est antecedens esse verum sine consequente,¹⁶⁴ ergo talis consequentie possibile est antecedens¹⁶⁵ esse verum; et omne quod potest esse verum est possibile, et sic impossibile potest esse verum; quod repugnat, quia propositionem impossibilem intelligimus propositionem que nullo modo potest esse vera.

Alia opinio declarat questionem per alium modum et presupponit aliqua. Primum est, quod licitum sit¹⁶⁶ arguere a primo ad ultimum, dummodo omnis consequentie intermedie fuerint bone. Presupponit¹⁶⁷ secundo, quod quelibet categorica potest inferre unam disiunctivam cuius ipsa est pars, scilicet sic: sor currit, igitur sor currit vel baculus stat in angulo. [O200rb] Presupponit tertio, quod a disiunctiva¹⁶⁸ cum opposito unius partis licitum est alteram¹⁶⁹ partem inferre.¹⁷⁰

Hiis premissis, hec opinio declarat hanc consequentiam: homo est asinus, igitur baculus stat in angulo. Homo est asinus, igitur homo est asinus vel baculus stat in angulo: ista consequentia tenet per secundam suppositionem.¹⁷¹ Tunc ultra: homo est asinus vel baculus stat in angulo, sed nullus homo est asinus,¹⁷² igitur baculus stat in angulo; hec secunda¹⁷³ consequentia tenet per tertiam suppositionem. Nunc accipio¹⁷⁴ primum antecedens et ultimum consequens: primum¹⁷⁵ antecedens fuit hoc, 'homo est asinus', et ultimum consequens fuit 'baculus¹⁷⁶ stat in angulo', et stat¹⁷⁷ hec conse-

163 probat]V probatur O

164 tunc... consequente]V om. O

165 antecedens]O om. V

166 sit]O est V

167 Presupponit]O om. V

168 a disiunctiva]O ad disiunctivam V

169 alteram]O aliam V

170 inferre] V infere O

171 per secundam]V propter primam (!) O

172 asinus]O assinus V

173 secunda]O om. V

174 Nunc accipio]O Non accipio V

175 Primum]O Quod primum V

176 baculus]O bachulus V

177 stat]V fiat O

quentia: homo est asinus, igitur baculus stat in angulo; ista consequentia tenet per primam suppositionem, que est arguere a primo ad ultimum.

Ista opinio fallit, primo,¹⁷⁸ quia hic¹⁷⁹ non arguitur a primo ad ultimum; et immo est, quia, dum arguitur a primo ad ultimum,¹⁸⁰ debet hoc observari, quod consequens consequentie intermedie fuerit antecedens in consequentia precedenti.¹⁸¹ Modo hoc non observatur. Quod patet, quia prima consequentia fuit ista: homo est asinus, igitur homo est asinus vel baculus stat in angulo,¹⁸² ecce quod consequens istius consequentie in¹⁸³ una disiunctiva debet esse, igitur antecedens secunde consequentie; et hoc non observabatur in processu [V77vb] huius oppinionis, quia dicebatur sic in illo processu: homo est asinus [O200va] vel baculus stat in angulo et nullus homo est asinus, igitur etc. Et ecce quomodo hic est error,¹⁸⁴ quia antecedens huius consequentie non fuit consequens precedentis, quia consequens precedentis fuit una simplex disiunctiva et antecedens huius¹⁸⁵ secundi processus est una copulativa constituta ex una disiunctiva tamquam ex una eius parte et una categorica tamquam ex alia.

Et etiam si ratio ista¹⁸⁶ valeret, eodem modo probarem quod ex neccessario potest quidlibet inferri. Et¹⁸⁷ sic probabo istam consequentiam esse bonam: deus est, igitur homo est asinus; et per consequens concludam quod ex vero sequetur falsum, quia arguam, ut dicta opinio arguebat: deus est, igitur deus est vel homo est asinus; hec consequentia tenet per unam¹⁸⁸ suppositionem. Et tunc ultra: deus est vel homo est asinus, et nullus deus est, igitur homo est asinus; hec consequentia tenet per aliam¹⁸⁹ suppositionem. Tunc capiatur primum¹⁹⁰ antecedens et ultimum consequens et sic habetur propositum.

178 primo]O om. V

179 hic]O om. V

180 et immo est quia dum arguitur a primo ad ultimum]O om. V

181 precedenti]O precedente V

182 angulo]O angulo V

183 in]O est V

184 error]V eror O

185 huius]O om. V

186 ista]O om. V

187 et]O om. V

188 unam]O eius add. V

189 aliam]O istam V

190 primum]O principale V

Est igitur aliter dicendum¹⁹¹ presupponendo duo. Primum est quod quelibet propositio possit¹⁹² se ipsam inferre; hoc est necessarium,¹⁹³ quia ab eodem ad idem valet consequentia. Secundum est quod quelibet categorica possit¹⁹⁴ elicere unam disiunctivam cuius est pars.

Hiis premissis deduco consequentiam et probō quod ista consequentia valeat, scilicet¹⁹⁵ tu es et tu non es, igitur homo est asinus; sive velis istam, scilicet¹⁹⁶ tu es et tu non es, igitur tu scribis.¹⁹⁷ Arguo sic: faciam hanc consequentiam, tu es et tu non es, igitur tu es vel tu scribis, et tu non es.¹⁹⁸ Hanc consequentiam probō esse bonam. Primo, eius prima pars [O200vb] antecedentis que est 'tu es' infert hanc partem consequentis que est 'tu es vel tu scribis'. Et hoc per unam suppositionem, quod est principium in loyca. Et secunda pars antecedentis que est 'tu non es' infert secundam partem consequentis, quia¹⁹⁹ sunt eedem. Ergo totum antecedens infert totum consequens. Est igitur hec consequentia bona: tu es et tu non es, igitur tu es vel tu scribis, et tu non es. Capiō tunc²⁰⁰ istud consequens et arguo sic: tu es vel tu scribis, sed tu non es, igitur tu scribis. Hec consequentia valet per hoc principium: a disiunctiva cum opposito unius partis ad alteram valet consequentia. Igitur a primo ad ultimum valet,²⁰¹ scilicet tu es et tu²⁰² non es, igitur tu scribis.

In qua consequentia de primo [V78ra] ad ultimum observantur²⁰³ omnes condiciones²⁰⁴ requisite et iste modus deducendi specialissimus reputatur et excellentissimus.

Et per hoc ponatur conclusio responsiva de quesito, scilicet quelibet consequentia cuius antecedens est impossibile est bona.

191 dicendum]V dictum O

192 possit]V posset O

193 necessarium]V necessarium O

194 possit]V potest O

195 scilicet]O sed V

196 scilicet]V om. O

197 et tu non ... scribis]O om. V

198 Arguo ... non es]O om. V

199 quia]V que O

200 Capiō tunc]O Tunc capio V

201 valet]O om. V

202 tu es et tu]O tu es vel tu scribis sed tu V

203 observantur]O observatur V

204 condiciones]O conditiones V

Et sequitur ex ista conclusione quod aliqua propositio potest inferre se ipsam et suam contradictoriam. Patet, quia sequitur: precise²⁰⁵ duplum est, igitur non precise duplum est. Et etiam sequitur quod precise duplum est. Et pro solutione cuiusdam rationis dico quod omnes due propositiones, quarum utraque est impossibilis vel altera, repugnant. Unde iste repugnant: precise duplum est et sor currit. Similiter iste due precise duplum est et precise duplum est. Et quod repugnent patet quia una infert oppositum alterius ut constat et sequitur²⁰⁶ quod aliqua propositio sibi ipsi repugnat. Unde hec propositio 'precise duplum est' sibi ipsi repugnat et contradicit [*O201ra*] quod²⁰⁷ patet quia²⁰⁸ ipsa infert oppositum sui ipsius. Et sequitur ultra quod aliqua propositio est sua contradictoria. Quod²⁰⁹ patet, quia hec propositio 'precise duplum est' sibi ipsi contradicit et per consequens est sua contradictoria.

Et hoc de secundo articulo.

<In Tract. IV, Quaest. 7, Art. 2>

Vigesima quarta questio:²¹⁰ *Utrum quelibet consequentia cuius consequens est necessarium*²¹¹ *sit bona.*

[...] Quantum ad secundum, sit hec prima²¹² suppositio: quelibet consequentia cuius oppositum²¹³ consequentis infertur oppositum antecedentis [*O202ra*] est bona; hec suppositio est principium in loyca.

Secunda suppositio: ad impossibile sequitur quidlibet; patet per precedentem questionem.

Tertia suppositio: si aliqua propositio est necessaria contradictoria eius est impossibilis. Ista²¹⁴ suppositio patet, quia²¹⁵ si non erit, igitur eius²¹⁶

²⁰⁵ precise]O precise V

²⁰⁶ et sequitur]V etc. O

²⁰⁷ quod]V et O

²⁰⁸ quia]V quod O

²⁰⁹ Quod]V om. O

²¹⁰ Vigesima...questio]V om. O

²¹¹ cuius...necessarium]V om. O

²¹² hec prima]V hoc sophisma

²¹³ oppositum]O om. V

²¹⁴ Ista]V Illa O

²¹⁵ quia]O om. V

contradictoria contingens vel necessarium. Non potest dici quod sit²¹⁷ necessarium,²¹⁸ quia tunc duo contradictoria essent vera et necessaria; nec²¹⁹ potest dici quod sit²²⁰ contingens, quia si sic, tunc possibile esset duo contradictoria fore simul vera. Deduco consequentiam, quia primo unum contradictorium est necessarium, cuius reliquum est contingens; sed propositio illa dicitur contingens, quia si sit falsa potest esse vera et e contra. Posito igitur isto in esse, ut quod hoc contingens sit verum, habeo quod utrumque contradictorium est verum.

Tunc, his suppositionibus premissis, sequitur conclusio respondens de questio sic: quolibet consequentia cuius consequens est necessarium est bona.

Quam conclusionem declaro ex premissis, quia sit *A* una talis consequentia cuius consequens²²¹ est necessarium. Tunc arguitur sic. Oppositum consequentis²²² istius²²³ consequentie infert oppositum antecedentis, igitur *A* consequentia est bona.²²⁴ Ista consequentia tenet ex prima suppositione. Sed probatur antecedens, quia huius consequentie *A*, *B* consequens est necessarium, ergo contradictorium huius consequentis est impossibile. Patet²²⁵ illa consequentia²²⁶ ex una suppositione que dixi,²²⁷ quod si aliqua propositio est necessaria, eius contradictoria²²⁸ est impossibile.²²⁹ Capiam igitur contradictorium consequentis et sit *C*. Tunc, cum²³⁰ *C* sit impossibile, *C* potest inferre oppositum antecedentis, quia ad impossibile sequitur quidlibet. Igitur *A* consequentia fuit consequentia²³¹ bona.

216 eius] *V* sua *O*

217 quod sit] *O* istud *V*

218 necessarium] *O* necessaria *V*

219 nec] *V* contra *O*

220 sit] *O* om. *V*

221 consequens] *O* om. *V*

222 consequentis] *O* antecedentis *V*

223 istius] *V* huius *O*

224 bona] *O* consequentia add. *V*

225 Patet] *O* om. *V*

226 consequentia] *O* tenet add. *V*

227 que dixi] *O* quia dixit *V*

228 contradictoria] *V* contradictorium *O*

229 impossibilis] *V* impossibile *O*

230 cum] *V* est *O*

231 consequentia] *O* om. *V*

Ex qua conclusione sequeretur, quod quelibet propositio mundi, quecumque sit illa, potest in bona consequentia inferre illam et²³² propositionem disiunctivam constitutam ex contradictoriis. Patet pro tanto, quia quelibet disiunctiva composita [O202rb] ex contradictoriis est necessaria; cum autem necessarium sequatur ad quidlibet, patet propositum ex predictis.²³³ Ex predictis sequitur ista²³⁴ consequentiam esse bonam: tu scribis, igitur²³⁵ non precise duplum est; patet, quia illa propositio 'non precise duplum est'²³⁶ est necessaria. Et sequitur aliud, scilicet²³⁷ quod quelibet condicionalis cuius consequens est necessarium est necessaria.²³⁸ Et potestis etiam dicere tale corollarium,²³⁹ quod²⁴⁰ quelibet condicionalis cuius antecedens est impossibile est [V79ra] necessaria; patet, postquam ad impossibile sequitur quidlibet.

Et hoc de secundo articulo.

²³² illam et]O om. V

²³³ ex predictis]O om. V

²³⁴ ista]O illa V

²³⁵ igitur]O tu add. V

²³⁶ est ... est]O om. V

²³⁷ aliud scilicet]V ad antecedens O

²³⁸ necessaria]O negativa V

²³⁹ corollarium]O corolarium V

²⁴⁰ quod]V om. O